Reading 13

Exercises on quotient spaces, II

We'll practice some point-set topology of quotient spaces using the idea of compactness.

Let X be a topological space, and let $\sim$ be an equivalence relation on X. Recall we have a quotient map $q : X \to X/\sim$ sending x to $[x]$.

Definition 13.0.1. The *quotient topology* on $X/\sim$ is the topology for which a subset U of $X/\sim$ is declared open if and only if $q^{-1}(U)$ is open.

Exercise 13.0.2. Let X be a topological space and let $\sim$ be any equivalence relation on X. Show that the quotien tmap is continuous.

Definition 13.0.3. Let X be a topological space. X is called *compact* if every open cover of X admits a finite subcover.

(That is, for any open cover $\{U_\alpha\}_{\alpha \in \mathcal{A}}$ of X, one can find a finite subset $\mathcal{B}$ of $\mathcal{A}$ so that $\{U_\alpha\}_{\alpha \in \mathcal{B}}$ is an open cover of X.)

Exercise 13.0.4. Let X_i be compact for $i = 1, \ldots, n$. Show that the disjoint union $X_1 \coprod X_2 \coprod \ldots \coprod X_{n-1} \coprod X_n$ is compact. (Recall that a subset U of the disjoint union is open if and only if $U \cap X_i$ is open in X_i for every i.

Exercise 13.0.5. Let X be compact, and let $\sim$ be any equivalence relation on X. Show that $X/\sim$ (with the quotient topology) is compact.

If you finish the above exercises, or if you feel they will be too easy, go back and complete the exercises from the previous reading.