25.6 Lab: Taylor polynomials

Exercise 25.6.1. Let $f(x) = \ln x$.

(a) Compute $f(1)$.

(b) Compute $f'(1)$.

(c) Compute $f''(1)$.

(d) Compute $f^{(3)}(1)$.

(e) Compute $f^{(4)}(1)$.

(f) Write the fourth degree Taylor polynomial $T_4(x)$ of $\ln x$ at $a = 1$.

(g) Use a calculator to compute $T_4(1.1)$.

(h) Have a calculator compute $\ln(1.1)$.

(i) How close are the two answers you got?

(j) Try comparing T_4 to ln by plugging in numbers like 1.2, 1.3, or 1.01, or 1.001. Would you say that T_4 does a good job of approximating ln?

Possible partial solution. Here is the degree 2 Taylor polynomial:

$$T_2(x) = 0 + (x-1) + \frac{-1}{2}(x-1)^2.$$

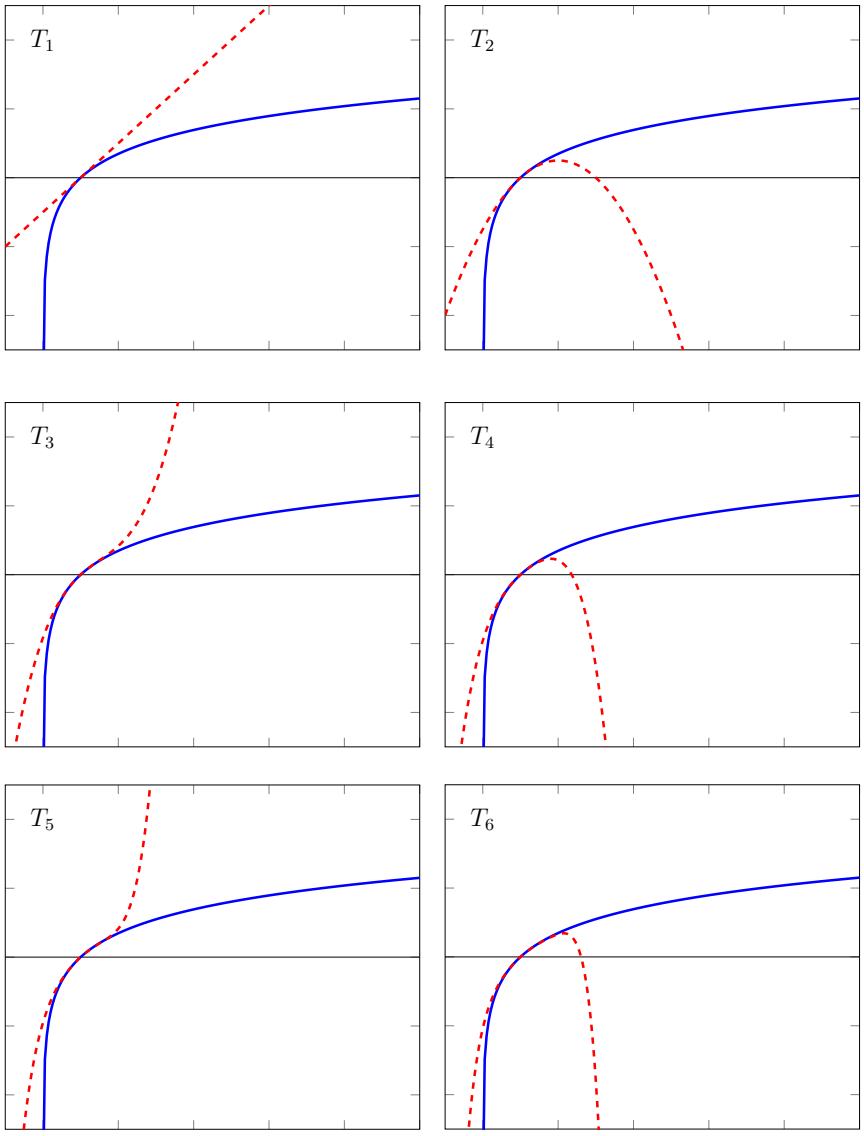
Here is the degree 4 Taylor polynomial:

$$T_4(x) = 0 + (x-1) + \frac{-1}{2}(x-1)^2 + \frac{1}{6}(x-1)^3 - \frac{1}{24}(x-1)^3$$

□

And, for fun, here are graphs of various Taylor polynomials for $\ln(x)$ at $a = 1$,

graphed along with $\ln(x)$:



Exercise 25.6.2. Let $f(x) = \sqrt{x}$.

(a) Compute $f(4)$.

(b) Compute $f'(4)$.

(c) Compute $f''(4)$.

(d) Compute $f^{(3)}(4)$.

(e) Compute $f^{(4)}(4)$.

(f) Write the fourth degree Taylor polynomial $T_4(x)$ of $\sqrt{x}$ at $a = 4$.

(g) Use a calculator to compute $T_4(5)$.

(h) Have a calculator compute $\sqrt{5}$.

(i) How close are the two answers you got?

(j) Try comparing T_4 to $\sqrt{x}$ by plugging in numbers like 4.1, 4.2, 4.3. Would you say that T_4 does a good job of approximating $\sqrt{x}$?

(k) Try comparing $T_4(5), T_4(6), T_4(7)$ to the square roots of 5, 6, 7. Would you say that T_4 does a good job of approximating $\sqrt{x}$ at these values of x?

Possible partial solution. Here is the degree 4 Taylor polynomial $T_4(x)$, not simplified (to expose a bit of the underlying work):

$$2+\frac{1}{2}\cdot\frac{1}{2}(x-4)+\frac{-1}{2}\cdot\frac{1}{2}\cdot\frac{1}{8}\cdot\frac{1}{2}(x-4)^2+\frac{-3}{2}\cdot\frac{-1}{2}\cdot\frac{1}{2}\cdot\frac{1}{16}\cdot\frac{1}{6}(x-4)^3+\frac{-5}{2}\cdot\frac{-3}{2}\cdot\frac{-1}{2}\cdot\frac{1}{2}\cdot\frac{1}{32}\cdot\frac{1}{24}(x-4)^4.$$

□