10.3 For next time

For next time, you should be able to do all the exercises in today's lecture. This involves carefully applying rules of derivatives to known expressions, and understanding what numbers are needed to determine a rate of change that depends on other quantities.

10.4 Lab exercises

Exercise 10.4.1. Hiro is on an awful amusement park ride – the kind that swings back and forth, way up in the air. Hiro's height can be modeled by the following function:

$$h_{\text{Hiro}}(t) = 50 + 5\sin(\frac{\pi t}{30})$$

where time is measured in seconds and h is measured in meters. At time $t = 30$ seconds into the ride, Hiro stupidly drops his phone. The fall of his phone can be modeled by the function

$$h_{\text{phone}}(t) = 50 - 9.8(t - 30)^2.$$

(a) Two seconds after Hiro dropped his phone, how far apart vertically are Hiro and his phone?

(b) Two seconds after Hiro dropped his phone, at what rate is the height between Hiro and his phone changing? What units are your answer?

Exercise 10.4.2. Two athletes, beginning at the same location, decide to run in perpendicular directions – A runs northward, while B runs eastward. Athlete A's position, as measured by the distance northward they have traveled, is modeled by the function

$$A(t) = 4\ln(t - 1)$$

while Athlete B's position, measured by the distance eastward travelled, is modeled by the function

$$B(t) = 5t.$$

Both functions take in a time t as measured in seconds, and output distance as measured in meters.

(a) Write a function $d(t)$ that tells you the distance between athlete A and athlete B at time t seconds, as measured in meters. (Hint: Pythagorean theorem.)

(b) Write a function $v(t)$ that tells you the rate at which the distance between A and B is changing at time t, as measured in meters per second.

(c) At time $t = 2$ seconds into the run, how quickly is the distance between A and B growing, in meters per second?

Exercise 10.4.3. A culture of bacteria is growing on a petri dish. At any time t, the bacteria are taking up a circular region on the petri dish, and the radius of this region is modeled by the following function:

$$r(t) = 8t$$

where t is in seconds and r is in micrometers.

At $t = 3$ seconds, how quickly is the area of the circular region changing, in units of micrometers-squared-per-second?

Exercise 10.4.4. A teardrop falls onto a lake, and the resulting ripple grows in radius. The radius can be modeled as a function of time as follows:

$$r(t) = 5e^{-3t}$$

where r is in centimeters and t is in seconds.

In terms of centimeters-squared-per-second, how quickly is the area enclosed by the ripple growing at $t = 2$ seconds?

Exercise 10.4.5 (An optimization problem)**.** You are running a subscription service, and your analysts have told you that the number of subscribers changes as a function of subscription cost as follows:

$$S(x) = 100000 + 20000(10 - x)$$

where x is the dollar amount cost of a yearly subscription (per subscriber), and S is the number of yearlong subscribers.

What should you make your yearly subscription costs to maximize revenue from subscribers?