Lecture 9

Exercises on max/min and so forth

9.1 Review

(a) The second derivative of f is the derivative of the derivative of f. We often denote the second derivative of f by f''.

(b) We say that a function f is *concave up* at x if $f''(x) > 0$. We say f is concave down at x if $f''(x) < 0$.

(c) We say x is an *inflection point* of f if f'' changes sign at x.

(d) We say that a number x is a *local maximum* of f if for every nearby point a, $f(x) \geq f(a)$. Likewise, we say f is a local minimum if for every nearby point a, $f(x) \leq f(a)$.

9.2 Some new terminology

We say that x is a *critical point* of f if $f'(x) = 0$.

We saw at end of class last time the following:

Theorem 9.2.1 (Second derivative test)**.** If x is a critical point and $f''(x) > 0$, then f is a local minimum.

If x is a critical point and $f''(x) < 0$, then f is a local maximum.

9.3 Exercises

Exercise 9.3.1. A company wants to use exactly 20 square centimeters of aluminum to create an aluminum can (with a top, a bottom, and the cylindrical side walls).

What is the largest-volume can that this company can make?

It may help to remember the following:

- The volume of a cylinder of radius r and height h is $\pi r^2 h$.
- The surface area of a cylinder of radius r and height h (the walls of the can, but without the top or bottom) is $2\pi r h$.
- The surface area of a circle of radius r (the top of the can, for example) is given by πr^2.

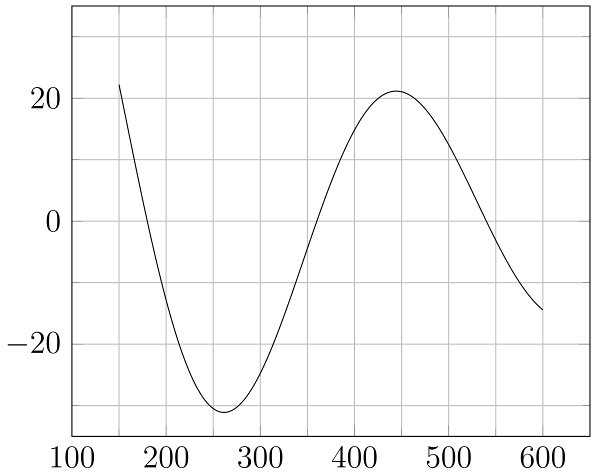
Exercise 9.3.2. The company decides that it would be unique to instead make a can whose top and bottom are not circular, but are squares.

What is the largest volume of a can (with square top and bottom, and rectangular sides) that this company can make out of 20 square centimeters of aluminum?

Exercise 9.3.3. Try both of the above exercises, but where now the company does not need to have a top to their cans (but still is using 20 square centimeters of aluminum).

Exercise 9.3.4. A train is equipped with a speedometer. The speedometer measures the velocity of the train (as estimated by how quickly the wheels of the train are rotating), and displays the velocity as positive if the train is moving northward, and negative if the train is moving southward.

Below is a graph showing the speedometer readings, in miles per hour. The horizontal axis is measured in minutes after 9 AM. So for example, at 60 units along the horizontal, the graph tells us the speed at 10 AM.



(a) When was the train moving fastest northward?

(b) When was the train moving fastest southward?

(c) During the timeframe depicted on this graph, can you tell what the local minima of the north-south position of this train were?

(d) During the timeframe depicted on this graph, can you tell what the local maxima of the north-south position of this train were?

(e) During the timeframe depicted on this graph, can you tell what the *global* minima and maxima of the north-south position of this train were?

9.4 For next time

For next class, I expect you to know the key terms from the topics we've covered this week (local minima, local maxima, inflection point, concave up, concave down, second derivative, critical point). I also expect you to be able to tell me what the second derivative test is. Make sure you are careful about what the *hypotheses* of the second derivative test are, and what the *conclusions* are.