Lab exercises: Concavity, local minima and maxima

Exercise 8.1.1 (Practice with second derivatives)**.** Compute the second derivatives of the following functions.

(a) $\cos(x)$

(b) $8x + 3$

(c) xe^x

(d) $\cos(x)e^x$

(e) $\ln(x)$

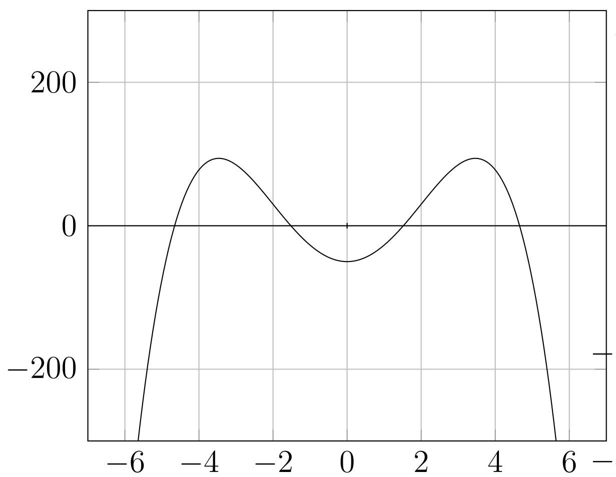
(f) e^{3x}

(g) $\ln(2x)$

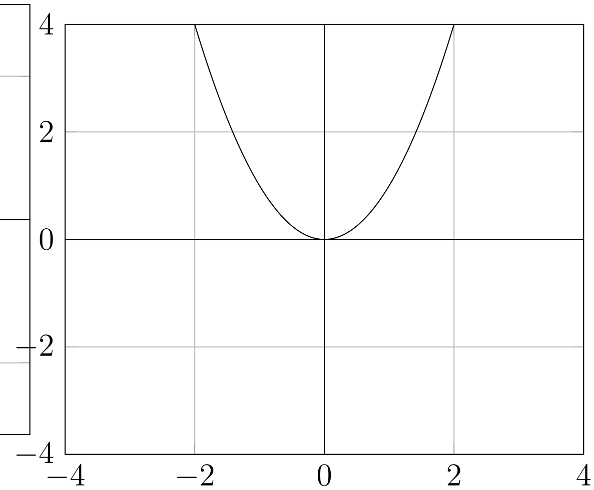
(h) $\ln(\sin(x))$

(i) $\sin(x)\cos(x)$

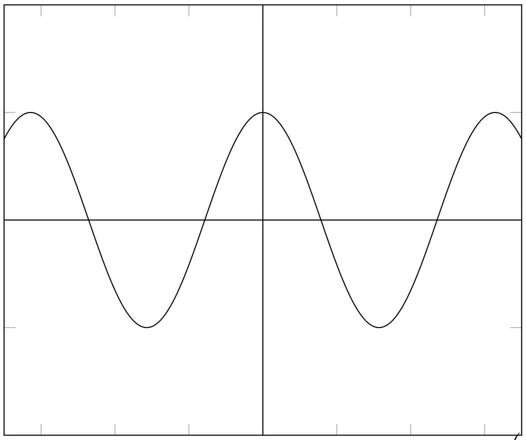
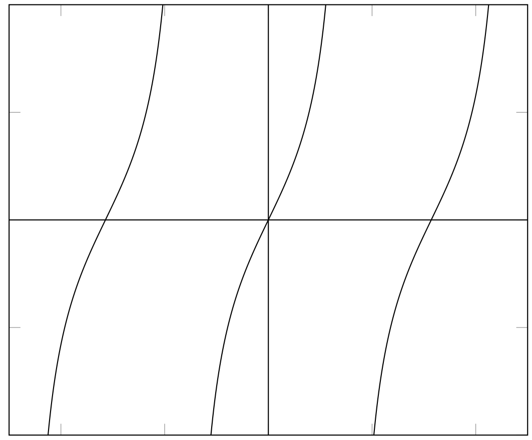
Exercise 8.1.2. For each of the functions graphed below, Shade in **bold** where the graph of the function has positive second derivative.*Draw a dot* at every inflection point.



(a)



(b)

(c)  (d) 

Exercise 8.1.3. Consider the function $f(x) = \frac{1}{3}x^3 - x$.

(a) Find all the points x at which $f'(x)$ is equal to zero.

(b) For each of these points, using the second derivative, determine which ones are local minima and which ones are local maxima. (We only went over this very quickly in class, so you might want to look at the course notes.)

(c) What are the values of $f(x)$ at the points you found?

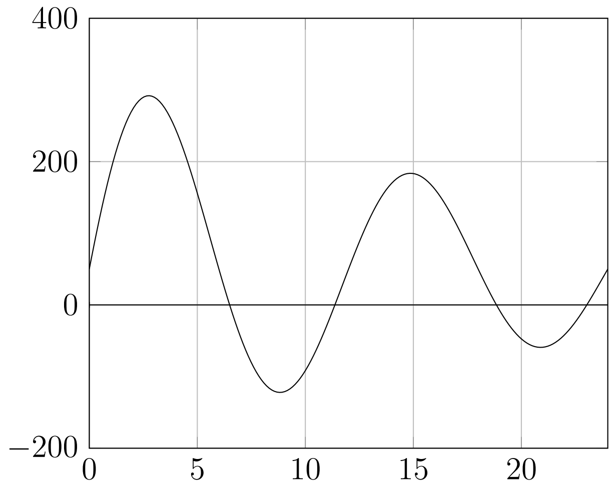
(d) Does f have any global maxima or minima?

Exercise 8.1.4 (Word problem)**.** You are an alpaca farmer. You want to enclose your alpaca in a rectangular plot of land using some fencing. Due to city regulations, one side of your rectangular plot must be fenced by fencing be made of pine, which costs 15 dollars for every foot of fencing. You want to make the three other sides of your rectangular plot be fenced by metal fencing, which costs 50 dollars for every foot of fencing.

And, for reasons of animal welfare, you want your plot of land to be exactly 10,000 square feet.

In feet, what should the length of the pine fencing be to minimize the cost of your fencing?

Exercise 8.1.5 (Word problem)**.** You are in charge of monitoring the water in a bay. You get live readings of the rate at which water is escaping the bay, as detected by some turbines' spin rates. Below is the graph showing the water's escape rate, as measured in acre-feet per hour.[2] When the escape rate is positive, water is escaping the bay (and the turbines spin one way). When the escape rate is negative, water is entering the bay (and the turbines spin the opposite way).



Here, the horizontal axis is being measured in hours starting from 8 AM on February 13th. Based on your readings of this graph:

(a) Estimate the time(s) at which water was escaping the bay fastest. (Your answer should not be something like "3" but an actual time, like 11 AM.")

(b) Estimate the time(s) at which water was entering the bay fastest.

(c) Estimate the time(s) on this graph at which the amount of water in the bay attained a local maximum.

(d) Estimate the time(s) on this graph at which the amount of water in the bay attained a local minimum.

[2] Acre-foot is a unit of volume, used commonly in the U.S. when measuring water in dams; though here we are using it for water in bays. An acre-foot is the amount of water that is required to cover one foot above a one-acre plot.