Lab exercises: Practice with product and quotient rules

Exercise 7.3.1 (Practice with product rule)**.** Compute the derivatives of the following functions.

(a) xe^x

(b) x^2e^x

(c) $\cos(x)e^x$

(d) $\sin(x)e^x$

(e) $x\ln(x)$

(f) $x^2\ln(x)$

(g) $\cos(x)\ln(x)$

(h) $\sin(x)\ln(x)$

(i) $\sin(x)\cos(x)$

(j) $\ln(x)e^x$

(k) $(3x^2+7)\sin(x)$

(l) $(\sin(x)+\cos(x))(x^3-9x)$

Exercise 7.3.2 (Practice with many rules at once)**.** Compute the derivatives of the following functions.

(a) $\ln(x^3e^x)$

(b) $e^{\cos(x)}\ln(x)$

(c) $\ln(x)\ln(\cos(x))$

(d) $e^{x\sin(x)}$.

Exercise 7.3.3 (Practice with quotient rule)**.** Compute the derivatives of the following functions.

(a) $\frac{1+x}{1-x}$

(b) $\frac{e^x}{x}$

(c) $\frac{\ln(x)}{x}$

(d) $\frac{\sin(x)}{\cos(x)}$

(e) $\frac{\cos(x)}{\sin(x)}$

(f) $\frac{1}{x}$

(g) $\frac{1}{\sin(x)}$

(h) $\frac{e^x-e^{-x}}{e^x+e^{-x}}$

Exercise 7.3.4 (Using product or quotient rule to generalize the power rule)**.** This is a fun one.

(a) Compute the derivative of x^{-3} in two ways: (i) using the quotient rule, and (ii) cleverly using the product rule (Hint: $x^3x^{-3} = 1$. Take the derivative of both sides.) How does your answer compare to what would happen if you applied the power rule to x^{-3}?

(b) Compute the derivative of x^{-7} in two ways: (i) using the quotient rule, and (ii) cleverly using the product rule. How does your answer compare to what would happen if you applied the power rule to x^{-7}?

Exercise 7.3.5 (Word problem: Density. You'd see something like this in astronomy, cosmology, or astrophysics)**.** A blob in outer space is forming. Its volume at time t is given by $V(t)$, where V is in cubic parsecs[1], and t is in years. The mass of the blob is given by $M(t)$, where M is in solar masses[2].

(a) Write an expression involving $V(t)$ and $M(t)$ that tells us the density of the blob at time t. (Remember that the density of something its how much mass it has per unit volume.)

(b) Write an expression involving $V(t), M(t), V'(t)$ and $M'(t)$ telling us the rate at which density is changing at time t. In terms of cubic parsecs, years, and solar masses – what are the units of your expression?

(c) Hiro models volume and mass of this blob by $V(t) = 100t$ and $M(t) = \ln(t)$. At $t = 10$ years, what is the rate at which the density of this blob is changing? Make sure you include the appropriate units.

Exercise 7.3.6 (Challenge problems)**.** For every function f below, think of a function F for which $F' = f$.

(a) $\frac{1}{x}$

(b) $\frac{1}{x+1}$

(c) $\frac{1}{3x}$

(d) $\frac{1}{3x+1}$

(e) $\frac{x}{x+1}$ (This one is very hard without being very clever.)

(f) $\frac{x}{x^2+1}$

[1] A parsec is a unit of distance. It is about 3.26 light-years. Note that, confusingly, light-year is a unit of distance.

[2] A solar mass is about 2×10^{30} kilograms.