Lab exercises: Derivatives of inverse functions, exp and ln

Exercise 0.0.1 (Practice with derivatives)**.** Compute the derivatives of the following functions.

(a) e^x

(b) $3e^x$

(c) e^{3x}

(d) e^{3x+5}

(e) $\ln(x)$

(f) $3\ln(x)$

(g) $\ln(3x)$

(h) $\ln(3x+5)$

(i) e^{x^2+8}

(j) $\ln(\cos(x))$

(k) $e^{\sin(x)}$

(l) $e^{\cos(x)}$

Exercise 0.0.2 (Practice with derivatives)**.** Compute the derivatives of the following functions.

(a) $\ln(\cos(x)^2 + \sin(x)^2)$

(b) $e^{\cos(x)^2+\sin(x)^2}$

(c) $\cos(e^x) + \ln(x^3 - \sin^2(x))$

(d) $(\ln(3x))^3 + (e^{\sin(x)})^2$.

Exercise 0.0.3 (Using inverses to compute more derivatives)**.** This is a fun one.

(a) Let $f(x) = x^3$ and $g(x) = x^{1/3}$. Compute $f(g(x))$ and $g(f(x))$.

(b) Using the chain rule cleverly[1], tell me $g'(x)$.

(c) Let $f(x) = x^4$ and $g(x) = x^{1/4}$. Using the chain rule cleverly, tell me $g'(x)$.

(d) Let $f(x) = x^5$ and $g(x) = x^{1/5}$. Using the chain rule cleverly, tell me $g'(x)$.

(e) Suppose n is some positive integer[2]. Can you tell me the derivative of $g^{1/n}$?

(f) How does your answer compare with the "power law" you already knew?

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[1] The same way we discovered the derivative of $\ln(x)$ once we knew the derivative of e^x.

[2] such as 2, 3, 4, 5, 6, 7, 8, 13, 1002, ...

Exercise 0.0.4 (Word problem: Population growth)**.** When a new species is introduced into an ecosystem, we sometimes use exponential functions to predict the population growth. Below is a function modeling the population growth of a family of 10 cockroaches who have found a new home.

$$P(t) = 10e^{\frac{\ln(2)}{5}t}$$

Here, P is in units of cockroaches, and t is in units of weeks.

(a) What are the units that $P'(t)$ is measured in?

(b) According to this model: At $t = 0$ weeks into the cockroaches' stay, at what rate is the population growing?

(c) According to this model: At 1 week into the stay, at what rate is the population growing?

(d) According to this model: At 2 weeks into the stay, at what rate is the population growing?

(e) According to this model: How many cockroaches will there be in this home after 3 weeks? (This requires no calculus!)

Exercise 0.0.5 (Challenge problems)**.** For every function f below, think of a function F for which $F' = f$.

(a) $2x$

(b) x

(c) $3x^2 - 7x + 9$

(d) $\sin(x) + \cos(x)$

(e) $\frac{1}{3}\sin(x) + 4\cos(5x)$

(f) $(x - 2)(x - 3)$

(g) $\frac{1}{11}(2x - 9)(x^2 - 9x)^{10}$

(h) $(\sin(x) + e^x)(-\cos(x) + e^x)^{13}$

(i) $xe^{(x^2)}$

(j) $e^x \sin(e^x)$