Lab exercises: Chain rule, exp and ln review

Exercise 0.0.1 (Practice with the chain rule). Compute the derivatives of the following functions.

(a) $\sin^2(x)$ [1]

(b) $\sin(x^2)$

(c) $\sin(x^2 + 3x + 1)$

(d) $\sin^2(x) + \cos^2(x)$

(e) $(\cos(x) + \sin(x))^4$

(f) $(x^2 + 2x + 1)^5$

(g) $(x^3 + x - 2)^8$

(h) $(x^8)^8$

(i) $\sin(\cos(x))$

(j) $\sin(x^2 + \cos(x^3))$

(k) $\cos(\cos(\cos(x)))$

(l) $\sin(\cos^2(x))$

Exercise 0.0.2 (Practice with exponentials and logarithms). Compute the following numbers. (None of your answers will involve e or ln.)

(a) $e^{\ln 3}$

(b) $e^{2\ln 3}$

(c) $e^2 e^3 e^{-5}$

(d) $e(e^5)^{1/5}$

(e) $\ln(1)$

(f) $\ln(e^0)$

(g) $\ln(e^3)$

(h) $\frac{1}{4}\ln(e^5)$

(i) $\ln(e) - 2$

(j) $\ln(e) + \ln(\frac{1}{e})$

(k) $\ln(e) + e^{-1}\ln(e^e)$

(l) $\ln(e^5) - e^{\ln 2}$

(m) $e^{(e^{\ln 2})}e^{-2}$

Exercise 0.0.3 (Word problem: Oscillatory motion). Hiro is pacing the room back and forth, between the door and the window. The following function describes Hiro's distance from the door at time t, where t is measured in seconds and d is measured in meters:

$$d(t) = 3 + 3\sin(\frac{2\pi}{2.8}(t - 0.7)).$$

(a) How far is Hiro from the door at $t = 0$ seconds?

(b) How quickly is Hiro moving at $t = 0$ seconds? (Make sure you state the units for your answer.)

(c) How far is Hiro from the door at $t = 0.7$ seconds?

(d) How quickly is Hiro moving at $t = 0.7$ seconds?

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[1] Remember that "$\sin^2(x)$" is lazy – but common – notation for "$(\sin(x))^2$".

(e) How far is Hiro from the door at $t = 1.4$ seconds?

(f) How quickly is Hiro moving at $t = 1.4$ seconds?

(g) Why is Hiro pacing? (Not a math question; creative answers encouraged.)

Exercise 0.0.4 (Word problem: Air pressure)**.** The air pressure P at a height h above sea level can be estimated by the following function:

$$P(h) = 100 \times (1 + \frac{1}{32}h + \frac{h^2}{2 \times (32)^2}).$$

P is measured in a unit called "Pascals," and h is measured in kilometers.

(a) According to the above estimate, at (0 kilometers above) sea level, what is the air pressure? (Your answer should be in Pascals.)

(b) What is the air pressure at 1 kilometer above sea level? You can leave your answer as a fraction (no need for a calculator).

(c) What units should the derivative $\frac{dP}{dh}$ have?

(d) At sea level, what is the rate at which air pressure is changing per kilometer of height?

(e) At 1 kilometer above sea level, what is the rate at which air pressure is changing per kilometer of height?

Hiro has created a terrifying amusement park ride – it takes you up and down repeatedly between 0 and 2 kilometers above sea level. When you are on this ride, your height can be modeled by the following function:

$$h(t) = 1 + \sin(\frac{\pi}{120}(t - 60))$$

where t is measured in seconds and h is measured in seconds.

(f) In what units is $h'(t)$ measured?

(g) When you are on this ride, how quickly are you moving at $t = 0$?

(h) When you are on this ride, how quickly are you moving at $t = 60$ seconds? (How many minutes into the ride are you at this point?)

(i) At $t = 0$ seconds, how quickly is air pressure changing for you? (Your answer should be in Pascals per second.)

(j) At $t = 60$ seconds, how quickly is air pressure changing for you?