Lab: Computing derivatives by hand

There is one new vocabulary word I want to use in today's lab: *Derivative.*

Definition 2.0.4. Let f be a function and choose a number x. If there is a number which the difference quotient

$$\frac{f(x+h)-f(x)}{h}$$

approaches as h goes to zero, we call this number the *derivative of f at x.*

Lab Problem 1 (Lines)**.** Let $f(x) = 3x + 2$.

(a) What does the graph of f look like?

(b) Based on your previous answer, and based on the equation of f, what do you expect the slope of *any* secant line to be?

(c) Let $x = 2$. What is the value of the fraction

$$\frac{f(x+h)-f(x)}{h}$$

for different values of h? (Does it even depend on h)? What value does it take as h approaches zero?

(d) Do the same problem as above, but with $x = 3$.

(e) For this function—$f(x) = 3x+2$—does the derivative of f at x depend on which value of x you choose?

Lab Problem 2 (Parabolas)**.** Now let $g(x) = 5x^2$.

(a) Let $x = 2$. Does the fraction

$$\frac{g(x+h) - g(x)}{h}$$

simplify when h does not equal zero? What value does it take as h approaches zero? In other words, what is the derivative of g at 2?

(b) Do the same problem as above, but with $x = 3$.

(c) Do the same problem as above, but with $x = 4$.

(d) Do you notice a relationship between the derivative of g at x, and x? Put another way, is there a formula that takes x as an input, and outputs the derivative of g at x?

Lab Problem 3 (Parabolas again)**.** Now let $q(x) = 5x^2 + 9$. (How is this function different from the previous function?)

(a) What is the derivative of q at $x = 2$?

(b) Do the same problem as above, but with $x = 3$.

(c) Do the same problem as above, but with $x = 4$.

(d) Do you notice a relationship between the derivative of q at x, and x? Put another way, is there a formula that takes x as an input, and outputs the derivative of q at x?

(e) How does the derivative of q relate to the derivative of g? How are q and g themselves related?

(f) What if we had added 5, or 2, or π, or one million to g?

Lab Problem 4 (Adding functions)**.** Now let $p(x) = 5x^2 + 3x + 2$. (How does this compare to the first two functions you studied—f and g?)

(a) What is the derivative of p at $x = 2$?

(b) Do the same problem as above, but with $x = 3$.

(c) Do the same problem as above, but with $x = 4$.

(d) Is there a formula that takes x as an input, and outputs the derivative of p at x?

(e) How does the derivative of p relate to the derivatives of g and q?

Lab Problem 5 (Cubics)**.** Let $r(x) = x^3$.

(a) What is the derivative of r at $x = 2$?

(b) Do the same problem as above, but with $x = 3$.

(c) Do the same problem as above, but with $x = 4$.

(d) Is there a formula that takes x as an input, and outputs the derivative of r at x?