Commitment to being a good educator

This is a math class. And I will speak, repeatedly, of what it means to be a mathematician. Part of my job this semester is to show you what it is to be a mathematician—in practice, through knowledge, and with confidence.

But let me state the obvious, which is that you are human beings first. And what I care much more about, much more than the math, is that you are able every day to walk upright as human beings.

If anything in your life is preventing you from learning; if not passing this class delays your life in anyway; if I am doing anything that prevents you from learning with dignity; that is a problem we should solve *together*, and you should not have to shoulder it alone.

So as I commit to being a good educator for each and every one of you, I want you to commit to tell me if I am not being a good educator. Your commitment will make my commitment a lot less empty.

Notes.

1. I use the verb “walk;” I do not mean this literally. I mean for you to feel upright however you navigate life.

2. The reason that some outside-of-class factors can be a problem to be solved “together”: For me to assess and aid in your learning, I need to contextualize it.

Lecture 1

Slopes and Introductions (our first Tuesday lab was conducted as a lecture)

Welcome to class! See syllabus for detailed logistical information. For now, you should know:

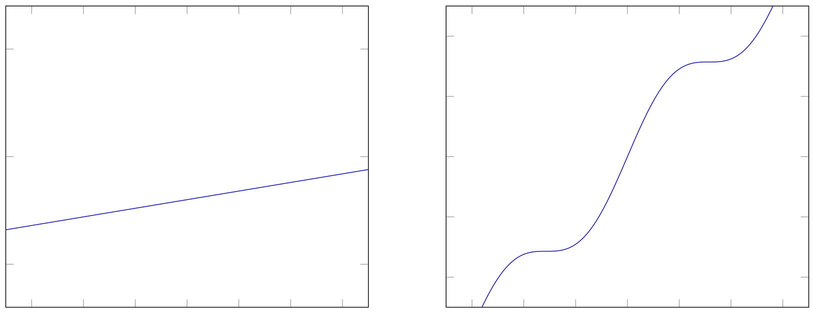
1. You are expected to come to every class and every lab. Groupwork and discussion is a large part of your learning for this class.

2. Every lecture, you will have a small quiz. What will be on the quiz? See the section "For next lecture" at the end of each day's class notes.

1.1 Lines

Question: What's the difference between curves and lines?

In everyday life—if you're not a mathematician, nor in a math class—we may not think of any difference between a curve and a line. But as a mathematical term, a line is always a *straight line*, and it is always infinitely long. Roughly speaking, it's the shape you can draw with a (long, long, long) ruler. A curve, on the other hand, is anything you can draw on a sheet of paper without ever lifting your writing utensil. And—it's in the name—it can be *curved* (not straight).



Above, you will see an example of a line on the left, and an example of a curve on the right. *Every line is an example of a curve, but not every curve is a line.*

Here is a fact about lines that you might remember:

> If you choose two different points on the plane, there is a line that goes through those two points. Moreover, there is exactly *one* line that goes through those two points.

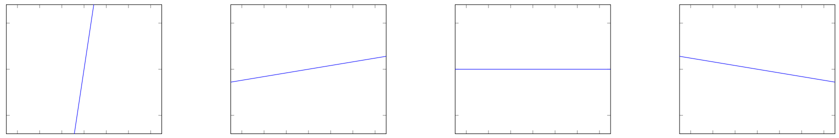
1.2 Slopes

Every line has a number called a *slope* associated to it. There are three lenses through which I'd like you to think about slopes (and in fact, about all major ideas we see in this course). These are the lenses of Intuition, Calculation, and Interpretation.

Remark 1.2.1. The calculational lens usually dominates how people think of "math." But math's true value to your education is its ability to challenge how you think – we will realize there are intuitions for things you'd never even though about, and that math can capture phenomena in real life in a way that expands our knowledge of the world around us. This is why I will emphasize the Intuition and the Interpretation of important mathematical ideas.

1.2.1 The intuition for slopes

Informally, slope measures how "tilted" a line is. If the slope is zero, the line is flat. If the slope is negative, the line is tilted downward. If the slope is positive, the line is tilted upward.



Above, you see pictures of lines of various slope. The leftmost line has a very large, positive slope. (The slope is so large, the line almost looks vertical.) The horizontal line has slope zero. The rightmost line has negative slope.

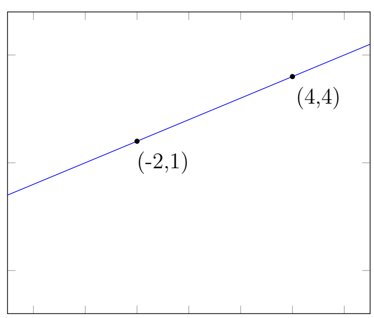
Remark 1.2.2. This is meant to be reminiscent of the usual use of the word "slope" in everyday life. You might have learned from previous classes that "positive slope" is the same thing as "uphill," while "negative slope" is the same thing as "downhill" as you move from left to right.

1.2.2 Calculating slope

As stated before, the slope of a line is a number. How do we calculate it?

To calculate the slope of a line: Choose two points on the line. Divide the vertical *rise* between the points by the horizontal *run* of the points. You may have seen this in a past math class as "rise over run."

Example 1.2.3. Below is the graph of a line.



The points $(-2, 1)$ and $(4, 4)$ are on this line. What is the slope of this line?

Answer: The "vertical rise" between the two points is the vertical difference between the two points. The point $(4, 4)$ has vertical coordinate 4, while the point $(-2, 1)$ has vertical coordinate 1, so the rise is

$$\text{rise} = 4 - 1 = 3.$$

The "horizontal run" between the two points is the horizontal difference between the two points. The point $(4,4)$ has horizontal coordinate 4, while the point $(-2,1)$ has horizontal coordinate -2, so the run is

$$\text{run} = 4 - (-2) = 4 + 2 = 6.$$

Then the slope is given by rise over run, so

$$\frac{\text{rise}}{\text{run}} = \frac{3}{6} = \frac{1}{2}.$$

In other words, the slope of this line is $1/2$. (Equivalently, the slope is given in decimals as 0.5.)

Remark 1.2.4. Above, I always measured *from* the point $(4,4)$ to the point $(-2,1)$ when measuring rise and run. You could have measured it the other way: From $(-2,1)$ to $(4,4)$. You'll get the same answer, so long as you are *consistent* about measuring *both* rise and run that way. For example, the rise is given by

$$1 - 4 = -3$$

and the run is given by

$$-2 - 4 = -6.$$

So rise over run is

$$\frac{-3}{-6} = \frac{1}{2}.$$

Tip 1.2.5. By the very definition of slope, you have to be comfortable with division when calculating slopes. This means *you will have to be comfortable with fractions*! If you find at any point that fractions are giving you trouble, don't despair: Just practice, practice, practice with fractions.

You may have also heard of slope as "change in y over change in x." Indeed, the "rise" between two points is the measure of how the y value changes between the two points, and the "run" is the measure of how the x value changes. So

$$\frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} \tag{1.2.1}$$

Make sure when you see a formula like (1.2.1), you know how to interpret it. Here, the above formula makes *no sense* unless you know that you are talking about two points on the x-y plane – the formula is referring to the rise and run between these two points (and the change in y and change in x between these two points).

1.3 Interpreting slope

A line can represent a *lot* of things. Accordingly, slopes can represent a lot of things. It all depends on what the line you're studying encodes.

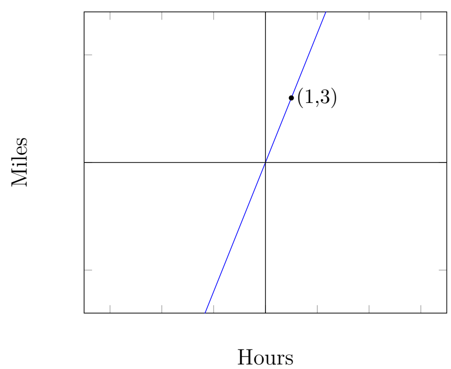
1.3.1 Constant velocity

This is an important example – the most important for us.

Dorothy is walking at a constant speed of three miles per hour. Let $f(t) = 3t$ denote the function that tells us how far Dorothy has walked (in miles) at time t (in hours).

For example, at $t = 0$, we see that $f(t) = 3 \cdot 0 = 0$, so Dorothy has walked 0 miles. At $t = 1$, $f(1) = 3 \cdot 1 = 3$, so Dorothy has walked 3 miles after 1 hour. At $t = 3$ hours, Dorothy has walked $f(3) = 3 \cdot 3 = 9$ miles.

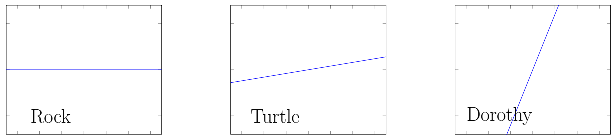
What does the graph of $f(t)$ look like?



Above, you see the graph of $f(t)$ in blue. (The t-axis, also known as the time axis, is horizontal.) For example, the point $(1, 3)$ is on this graph. What you immediately see is that the graph is a line!

In general, it turns out: *If something moves with constant velocity, the position-versus-time graph will always be a straight line.* In the above example, Dorothy moved with constant velocity, so her position-versus-time graph was a straight line.

Here are three position-versus-time graphs of three objects moving at constant velocity: A rock, moving at 0 miles per hour, a turtle, moving at 0.2 miles per hour, and Dorothy, moving at 3 miles per hour.



As you can see, *the faster something is moving, the steeper the line.* Well, we saw above that steep lines have large slopes, so we can conclude

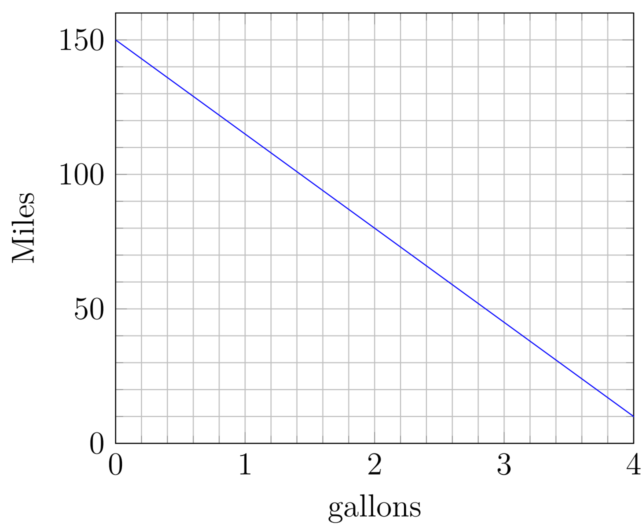
If a position-time graph is a line: The faster something is moving, the larger the slope.

That is, we witness a relationship between an object's speed, and the slope of its position-versus-time graph.

1.3.2 Rates and Ratios

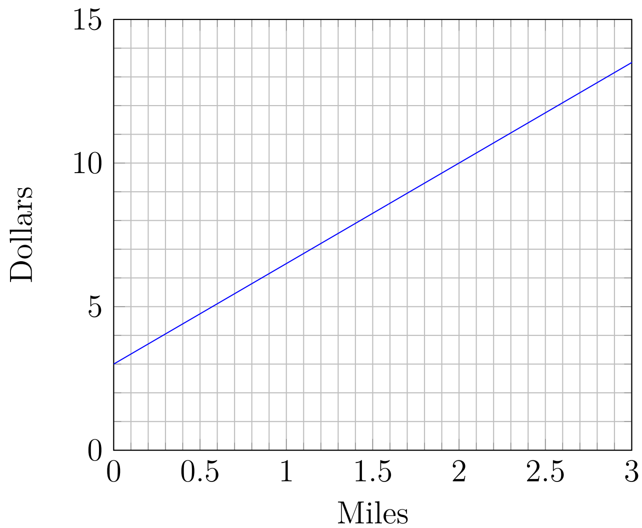
(We will do this in small groups.)

Example 1.3.1 (Miles per gallon)**.** Below is a graph showing (on the vertical axis) the mile marker of Hiro's car traveling down I-35, and (on the horizontal axis) the number of gallons of gas in the car's tank. The points on the blue line indicate what mile marker the car was at when the car had a particular amount of gas left in the tank.



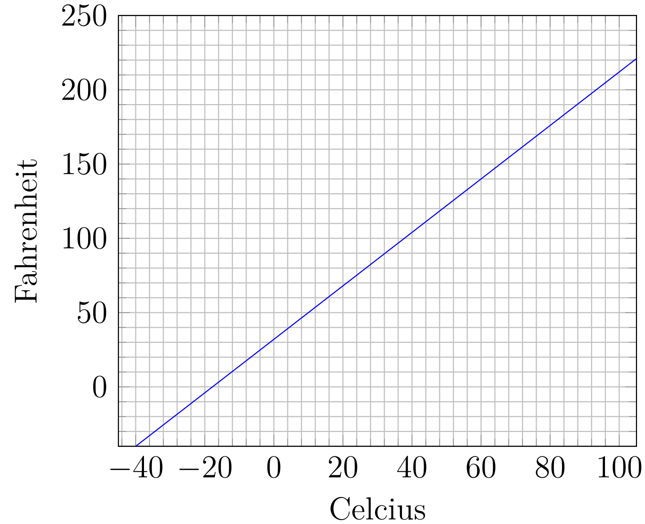
From this graph, can you estimate the fuel efficiency of Hiro's car? (That is, during the drive graphed, how many miles was Hiro's car using per gallon of gas?) What does this have to do with the slope of the above line?

Example 1.3.2 (Taxi fare)**.** Below is a graph showing (on the vertical axis) the taxi fare in dollars of the cab Hiro took in Manhattan, New York, the other week, and (on the horizontal axis) the number of miles the cab drove. The points on the blue line indicate what the taxi's fare meter showed when the cab had traveled a particular mileage.



From this graph, can you estimate the how much this New York cab was charging per mile? How many dollars per mile? What does this have to do with the slope of the above line?

Example 1.3.3 (Fahrenheit versus Celsius)**.** Below is a graph showing (on the vertical axis) the temperature in Fahrenheit, and (on the horizontal axis) the temperature in Celcius. The points on the blue line indicate what a thermometer would read in both Fahrenheit and in Celcius at a given temperature.



Using this graph, can you tell me: If the temperature drops by 5 degrees Celcius, how many degrees Fahrenheit does it drop by? What does this have to do with the slope of the above line?

The above examples hint at a general principle:

> *The slope of a line often represents the rate at which one quantity depends on another.*

Remark 1.3.4 (Units of measurement)**.** Notice that in all of our examples, the slope of the lines had natural *units* of measurement: Miles per hour, miles per gallon, dollars per mile, and degree fahrenheit per degree celsius. And these units are consistent with the units of the numbers we use when computing slope. For example: In Example 1.3.1, the "rise" between two points on the line measures the vertical distance between two points – i.e., something measured in miles. Meanwhile, the "run" between two points on the line measures the horizontal distance between two points – something measured in gallons. Dividing something measured in miles by something measured in gallons, we arrive at a number measuring miles per gallon.

1.4 Practice computing slopes

Given two points, be prepared to find the slope of the line between those two points.

Sometimes, we will call our two points P and Q.

Exercise 1.4.1. For each of the following pairs of points P and Q, find the slope of the line passing through P and Q.

1. $P = (1, 1)$, $Q = (2, 2)$.
2. $P = (1, 1)$, $Q = (4, 4)$.
3. $P = (1, 1)$, $Q = (7, 6)$.
4. $P = (1, 3)$, $Q = (7, 3)$.

The slopes are 1, 1, 6/5, and 0, respectively. Make sure you can explain to me how you compute these answers.

You will also want to know how to find the slopes of lines when they are given as an equation involving x and y.[1]

Exercise 1.4.2. Tell me the slopes of all the following lines:

(a) $y = 3x$.

(b) $y = 3x + 9$.

(c) $y = 3x - 10$.

(d) $y = 7x + 1$.

(e) $y = \frac{\pi}{3}x + 2$.

(f) $y = \frac{1}{2}(x - 9)$.

(g) $(y - 5) = 8(x - 2)$.

(h) $(y - \frac{1}{2}) = \frac{5}{7}(x - \frac{3}{4})$.

(i) 3y + 5x - 9 = 0.

The slopes are $3, 3, 3, 7, \frac{\pi}{3}, \frac{1}{2}, 8$, and $\frac{5}{7}$, and $-\frac{5}{3}$. Make sure you can explain to me how you compute these answers.

Because this is the first week of class, let me remind you to take a look at the Canvas course website. Either in the Assignments page, or on the main page of the course website, take a look at the upcoming homework. Make a note of their due dates. Get an early start.

[1] This is supposed to be "review" from your previous courses, so if you are not sure how to do this, make sure you dust off your old knowledge. Feel free to use TAs, Hiro, and your classmates as a resource if things is rusty.

1.5 For next time

Given two points P and Q, you should be able to compute the slope of the line passing through those two points. This was worked out in Example 1.2.3. There are many practice problems for you in Exercise 1.4.1.

Given the graph of a line that represents some real-world phenomenon, you should be able to tell me what the slope of that line represents. (You will want to understand Section 1.3.) You should also be able to tell me the *units* of the physical quantities represented by slope in a given situation. (See Remark 1.3.4.)