Proposition: R is an equivalence relation

Defn: Fix a topological space X.
$\pi_0 = X/x\sim x'$ iff $\exists$ a continuous path from x to x'
(also called the set of path connected components of X

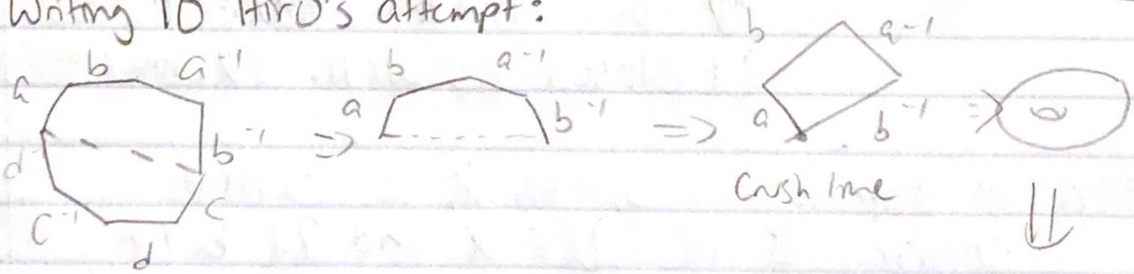
ex. $X \subset \mathbb{R}^2$

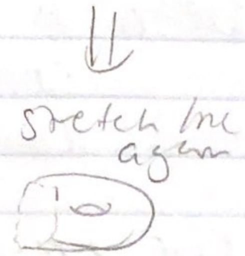
X =

In this example, $\pi_0(X)$ has 6 elements (6 eq classes)

Proposition: If $f: X \to Y$ is a homeomorphism then f induses a bijection $\pi_0(X) \xrightarrow{\cong} \pi_0(Y)$

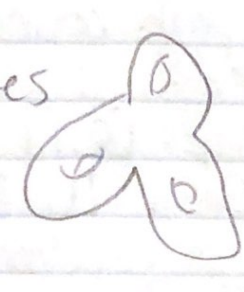
Writing 10 Hiro's attempt:



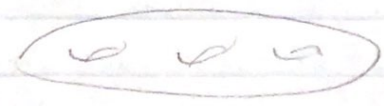


do the sam thing for other half to get

Dodecagon makes



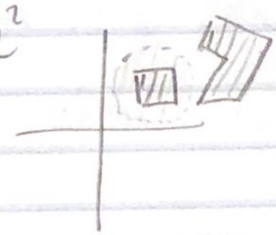
which is homeomorphic to



All factors of 4 work. 4·g gon makes genus g

Last time: path connectedness & π_1 rough.
Today: Connectedness

Let $A =$  $\in \mathbb{R}^2$
notice A is not path connected
By subspace topology $U \subset A$ is open iff
$\exists$ open $W \subset \mathbb{R}$ st $U = W \cap A$.

$\mathbb{R}^2$ 

Let $U =$ "square", $V =$ "Six sided fig."
Could find open ball in $\mathbb{R}^2$, W,
st $U = W \cap A$. So U open.
Additionally, could find some
$W \in \mathbb{R}^2$ st $V = W \cap A$. Thus
V open. But then $U \in V^c$ so
U closed as well. (V closed too)

Defn: A topological space X is called
connected if $U \subset X$ and U both
open and closed, then $U = \emptyset$ or X.

Our example above is <u>not</u> connected

<u>Proposition</u>: If X is path connected, then X
is connected.
Lemma needed to prove prop: $[0,1]$ is connected
pf of lemma: NTS Given $U \in [0,1]$ st U is
both open and closed, U is either empty or
$U = [0,1]$. (I) If $U \subset [0,1]$ is open and if $b \in U$
st $b \neq 0, 1$, then $\exists$ $\varepsilon > 0$ st $(b-\varepsilon, b+\varepsilon) \subset [0,1]$ -

(II) If $\mathcal{U} \subset [0,1]$ is closed then $\mathcal{U}$ contains a maximal element. (Heine borel & EVT)
So it is impossible for $\mathcal{U}$ to be closed, open, and not contain $[0,1]$.

Use lemma to prove prop (in notes)

Defn: A topological space is called disconnected if X is not connected

Proposition: The following are equivalent:
(1) X is disconnected
(2) $\exists$ two open subsets $\mathcal{U}, \mathcal{U}' \subset X$ s.t. $\mathcal{U} \cup \mathcal{U}' = X$ and $\mathcal{U} \cap \mathcal{U}' = \emptyset$