10/26

Last time: Metric Spaces!

Def: Metric on set X is a function $d: X \times X \to \mathbb{R}$.
such that:

① $d(x, x') = 0 \Leftrightarrow x = x'$

② $d(x, x') = d(x', x)$

③ $d(x, x'') \leq d(x, x') + d(x', x'')$

New Def: Let X and Y be metric spaces (a pair (X, d) when X is a set and d is a metric). A function $f: X \to Y$ is called an isometry if: ① f is a bijection

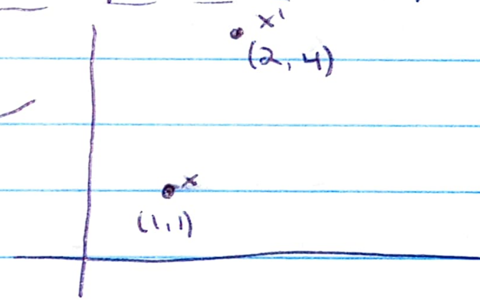
② $\forall x, x' \in X$, $d_Y(f(x), f(x')) = d_X(x, x')$

↳ doesn't necessarily mean the <u>same</u> metric but is equivalent

Ex: Let $X = Y = \mathbb{R}^n$. Define $d: X \times X \to \mathbb{R}$,

$(x, x') \mapsto \sqrt{\sum_{i=1}^{n} (x_i' - x_i)^2}$ and $d_Y: Y \times Y \to \mathbb{R}$,

$(y, y') \mapsto 3\sqrt{\sum_{i=1}^{n} (y_i' - y_i)^2}$



$d(x, x') = \sqrt{(2-1)^2 + (4-1)^2}$

$= \sqrt{1+9} = \sqrt{10}$

$d_Y(x, x') = 3\sqrt{10}$

Then the function $f: X \to Y$, $x \mapsto 3x$ is an isometry

If f is an isometry it is also continuous and exhibits a homomorphism