Questions of the Day:

- "well defined" is a term used to make sure that a formula makes sense

ex) $\bar{f}([x]) = f(x)$

$\bar{f}$([candice]) = candice's bday

$\bar{f}$([person x]) = person x's bday

this is not well-defined

$$\begin{tikzcd} \{\text{student}\} \arrow[r, "f"] \arrow[d] & \{\text{days}\} \\ \{\text{rows}\} \arrow[ur, dashed, "\bar{f}"'] & \end{tikzcd}$$

- often to verify that "$\bar{f}$ is well defined" is to show $x \sim x' \Rightarrow f(x) = f(x')$

note: in hw $\sin(2y)$ should be $\sin(2x)$

Today: Hausdorff-ness

motivation: we now have many topological spaces.

$$P \qquad \begin{array}{c} A \subset \mathbb{R}^n \\ \downarrow \\ A/\sim \end{array} \qquad X \times Y$$

Hausdorffness is a property (of a topological space) preserved under homeomorphisms — so it helps us distinguish spaces.

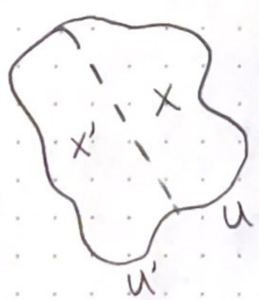
Def. (Hausdorff): Fix a topological space X. X is called hausdorff if $\forall x, x' \in X$ st $x \neq x'$, $\exists\, U, U' \subset X$ st

a) U, U' are open

2) $x \in U$ and $x' \in U'$

3) $U \cap U' = \emptyset$

ex)



or

ex) let $X = [1]$ w/ the Alexandroff topology

$X = \{0, 1\}$ $\qquad 0 \leq 1$

let $x = 0$, $x' = 1$

$U = \{0, 1\}$

$U' = \{1\}$

so $U \cap U' = \{1\}$

let U be an open set w/ 0 as an element

so, $0 \in U$ & $1 \in X$ st $0 \leq 1$, then $1 \in U$

note that $\mathcal{J} = \{\emptyset, \{1\}, \{0,1\}\}$

Corollary (immediate consequence): let $Y = \{0, 1\}$ with the discrete topology then Y is not homeomorphic to $X = [1]$

ex) any discrete space is hausdorff (given $x \neq x'$, let $U = \{x\}$, $U' = \{x'\}$ so $U \cap U' = \emptyset$)

examples of homeomorphic spaces:

D^2

basically a shape that was pinched (if you have a point in D^2 and then pinch the shape to change it, the point is still there but it moves)

examples of spaces not homeomorphic to D^2 (nor each other):

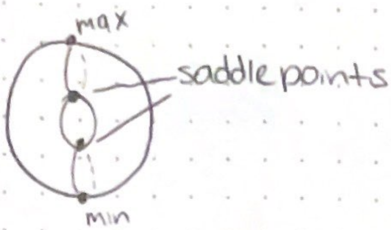
- a point

not path connected (while in D^2, if you remove a point you can still connect 2 points through another path)

D^1

$\mathbb{R}^3$

To show D^1 is not homeo to D^2, note that no matter what point you remove from D^2, the resulting shape is path-connected, but you can remove a point from D^1 so that the resulting shape is not path-connected.



This "2" is the Euler characteristic of the sphere. It turns out that for compact, path-connected, 2-dimensional, orientable shapes, the Euler characteristic completely classifies the shape up to homeomorphism.

homeomorphisms can change the # of saddle points

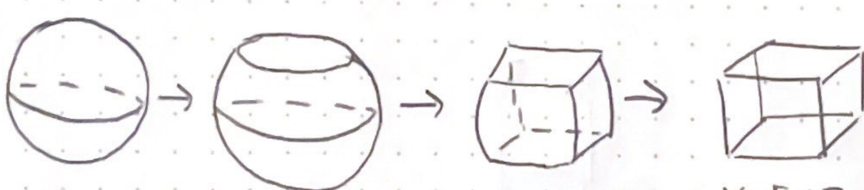
#max − #sadd − #min	2	2	2
#max	1	3	2
#min	1	2	1
#saddle	0	3	1

S^2

shows that they are homeomorphic

For the middle column, the sphere as drawn actually does not have the indicated number of max/min/saddles; challenge: draw a sphere embedding that does have the indicated number of 3 max, 2 min, and 3 saddles!

spheres are actually homeomorphic to cubes!



V−E+F	12
#vertices	8
#edges	12
#faces	8

# of faces of a cube is actually 6 :-)
(think of rolling dice)