Q: Difference "open" in $\mathcal{T}$? 9/19

" $\mathbb{R}^n$? $\longrightarrow$ has a standard topology : $\mathcal{T} = \{ U \subset \mathbb{R}^n \mid \forall x \in U, \exists r > 0 \text{ s.t. } \text{Ball}(x,r) \subset U \}$

" a poset P? $\longrightarrow$ has a Alexandroff topology : $\mathcal{T} = \{ U \subset P \mid \forall p \in U, \forall p' \in P \text{ s.t. } p' \geq p, p' \in U \}$

Compactness

Defn: Fix a set X, + a collection of subsets

$$A \to P(X), \quad \alpha \mapsto U_\alpha$$

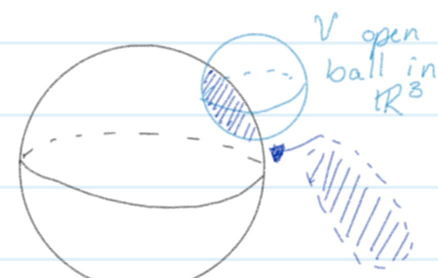
$\{U_\alpha\}_{\alpha \in A}$ is a cover (of X) if $\bigcup_{\alpha \in A} U_\alpha = X$.

If X is a topological space + each U_α is open, we say $\{U_\alpha\}_{\alpha \in A}$ is an open cover.

Ex/ Let X = the surface of the earth. Given an atlas, let A = the set of pages of the atlas. For each page α, let U_α be the position of X mapped on page α. Then $\{U_\alpha\}_{\alpha \in A}$ is a cover.

Further, each U_α depicts only open subsets of X.

Then $\{U_\alpha\}_{\alpha \in A}$ is an open cover.



$X = S^2 \subset \mathbb{R}^3$

Defn Fix a cover $\{U_\alpha\}_{\alpha \in A}$ of X. Then a collection $\{U_\beta\}_{\beta \in B}$ is a subcover of $\{U_\alpha\}_{\alpha \in A}$ if:

① $B \subset A$

② $\{U_\beta\}_{\beta \in A}$ is a cover of X.

Defn: A topological space X is compact if every open cover of X admits a finite subcover.

B is a finite set