Question:

"open" in $\mathcal{T}$

- $\mathbb{R}^n$ has a standard topology: $\mathcal{T} = \{ U \subset \mathbb{R}^n \mid \forall x \in U, \exists r>0 \text{ st } Ball(x,r) \subset U \}$
- A poset P has the Alexandroff Topology: $\mathcal{T} = \{ U \subset P \mid \forall p \in U, \forall p' \in P \text{ s.t } p' \geq p, p' \in U \}$.

9/19/23

Today: Compactness

Definition: Fix a set X, and a collection of subsets

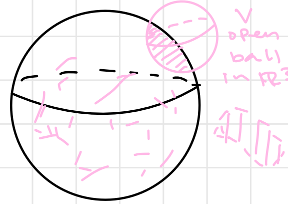
$A \longrightarrow P(X)$

$\alpha \longmapsto U_\alpha$

$\{U_\alpha\}_{\alpha \in A}$ is a cover (of X) if $\bigcup_{\alpha \in A} U_\alpha = X$.

If X is a topological space and each U_α is open, we say $\{U_\alpha\}_{\alpha \in A}$ is an open cover.

Ex: Let X = the surface of the earth. Given an atlas, let A = the set of pages of the atlas. For each page α, let U_α be the portion of X mapped on page α. Then $\{U_\alpha\}_{\alpha \in A}$ is a cover. If, further, each U_α depicts only open subsets of X. Then $\{U_\alpha\}_{\alpha \in A}$ is an open cover.



$X = S^2 \subset \mathbb{R}^3$

Definition: Fix a cover $\{U_\alpha\}_{\alpha \in A}$ of X. Then a collection $\{U_\beta\}_{\beta \in B}$ is a subcover of $\{U_\alpha\}_{\alpha \in A}$ if:

1.) $B \subset A$

2.) $\{U_\beta\}_{\beta \in B}$ is a cover (of X).

Definition: A topological space X is compact if every open cover of X admits a finite subcover. B is a finite set