DATE: September 7, 2023

$$[2] \times [2] \cong \{0,1,2\} \times \{0,1,2\} = \left\{ \begin{matrix} (0,0), & (1,0), & (2,0) \\ (0,1), & (1,1), & (2,1) \\ (0,2), & (1,2), & (2,2) \end{matrix} \right\}$$

$\uparrow$ isomorphism (bijection)

$f(x)=x^2$ $\quad \left[\begin{matrix} f: \mathbb{R} \longrightarrow \mathbb{R} \\ x \longmapsto x^2 \end{matrix} \right.$

$$\underset{\uparrow \text{ function}}{p}: \underbrace{\overbrace{S^3}^{\text{domain}}}_{} \longrightarrow \underbrace{\overbrace{\mathbb{C}}^{\text{complex \#}} \times \mathbb{R}}_{\text{co-domain}},$$

// $\mathbb{C} \cong \mathbb{R}^2$: $a+bi \longmapsto (a,b)$
$a+bi \longleftarrow (a,b)$

// $S^3 \subset \mathbb{R}^4 \cong \mathbb{R}^2 \times \mathbb{R}^2 \cong \mathbb{C} \times \mathbb{C}$ // $\quad p: S^3 \longrightarrow \mathbb{C} \times \mathbb{R} \cong \mathbb{R}^3$
$\quad S^3 \longrightarrow S^2$

Last time:

Open subsets of $\mathbb{R}^N$

Defn ⓐ: $U \subset \mathbb{R}^N$ is called <u>open</u> if U is a union of open balls.

Defn ⓑ: A subset U of $\mathbb{R}^N$ is called <u>open</u> iff:

$$\forall_x \in U, \ \exists_r \in \mathbb{R}, \ r > 0, \text{ such that } \mathrm{Ball}(x,y) \subset U$$

proposition: U satisfies ⓐ $\iff$ U satisfies ⓑ

Today: Given a subset $A \subset X$ recall that the complement of A is the set of all $x \in X$ <u>NOT</u> in A. Often, we write

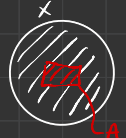
A^c for the complement (of A in X) in other words,

$$A^c := \{ x \in X : x \notin A \}$$

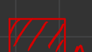
Notation: Sometimes we write:
$X \setminus A$ (or $X - A$)
to mean A^c

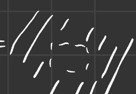
Ex:
$$\mathbb{Z} / \{0\} = \{0\}^c \ (\text{in } \mathbb{Z})$$
$$= \{\ldots -2, -1, 1, 2, \ldots\}$$

Ex:

- 

$\Rightarrow A^c =$ 

- 

$X = \mathbb{R}^2$
$\Rightarrow A^c =$ 

<u>Defn</u>: A subset $K \subset \mathbb{R}^N$ is called <u>closed</u> if K^c is open.

⚠ It is possible for some subsets to be both open & closed.