DATE: August 31, 2023 Unions & Intersections

write one for hiro.

freedom to choose

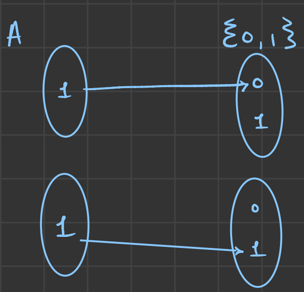
homework #1 : Exhibit a bijection between $P(A)$ and the set of all functions from A to the two elements set $\{0,1\}$.

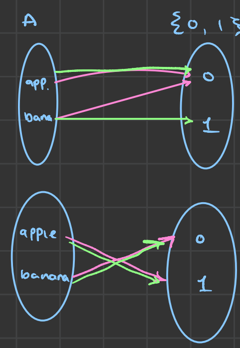
Compare between these two.

A bijection is a function from A to $\{0,1\}$. Domain $\longrightarrow$ Co-domain

"Every bijection has an inverse."

Ex: $A=\emptyset$ $A=\mathbb{N}$ $A=\{\text{apple}, \text{banana}\}$ $A=\mathbb{Z}$ $A=\{1\}$





Example:

$A=\{a,b,c\}$

$\{f : A \to \{0,1\}\} =$

f(a)	f(b)	f(c)
1	1	1
0	1	0
0	0	1
1	0	0
1	0	1
0	1	1
0	0	0
1	1	0

$A=\{a,b,c,d,e\}$

$B=\{\text{John}, \text{Jerry}, \text{Ron}\}$

//order of domains matter! //

$$\{f : A \longrightarrow B\} = 3^5$$

$$\{f : B \longrightarrow A\} = 5^3$$

Definition ≠ Explanation (A defn (of a term) is just a stand in for an Idea).

Definition ≠ Elaboration Ex: S^2

$S^2 := \{(x_0, x_1, x_2) \in \mathbb{R}^3 : x_0^2 + x_1^2 + x_2^2 = 1\}$ or S^2 is the set of elements in $\mathbb{R}^3$ having distance 1 from origin.

Fix a set X, and a set, and a set A and a function.

$$\begin{array}{ccc} A & \longrightarrow & P(A) \\ \rotatebox{90}{$\in$} & & \\ \alpha & \longmapsto & U_\alpha \end{array}$$

$\longrightarrow$ = denotes a function to another function.

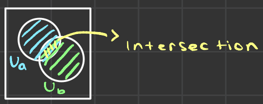
$\longmapsto$ = what it does to elements

<u>Notation</u>: We write $\{U_\alpha\}_{\alpha\in A}$ for this function.

<u>Definition</u>: The union of $\{U_\alpha\}_{\alpha\in A}$ is the set:

$$\bigcup_{\alpha\in A} U_\alpha := \{x\in X : \text{for at least one } \alpha\in A,\ x\in U_\alpha\}$$

1. $A = \{a, b\}$



The intersection of $\{U_\alpha\}_{\alpha\in A}$ is the set:

$$\bigcap_{\alpha\in A} U_\alpha := \{x\in X : \text{for every } \alpha\in A,\ x\in U_\alpha\}$$