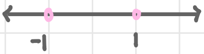
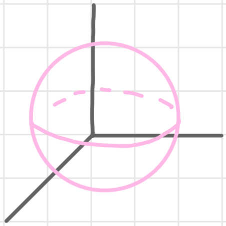
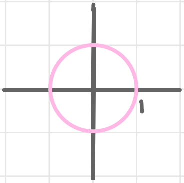
review:

S^0: 

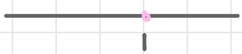
2 points

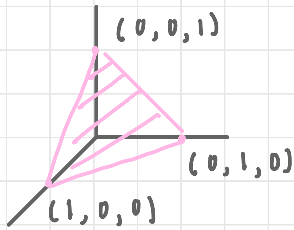
S^2: 
unit sphere

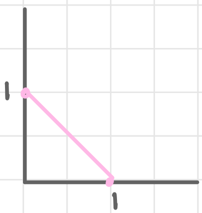
S^1: 

unit circle

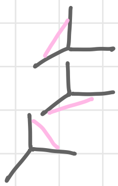
S^3, S^n

Δ^0: 

Δ^2: 

Δ^1: 

note: $\Delta^2 \cap \{x_1 = 0\}$
$\{x_2 = 0\}$
$\{x_3 = 0\}$

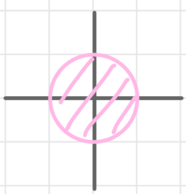


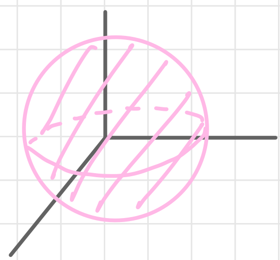
$\Delta^n \subset \mathbb{R}^{n+1}$
↳ set each dif coordinate to 0 to help on EC.

D^0: * a point

D^1:

$\mathbb{R}$, $S^0 \subset D^1$ aka S^0 are the bounds of D^1

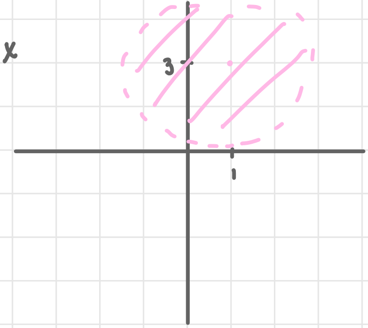
D^2: 
S^1 is the boundary (circle)
$S^1 \subset D^2$

D^3: 

$D^n = \{(x_1, x_2, \ldots, x_n) \mid \sum_{i=1}^{n} x_i^2 \leq 1\}$

notation: given $x \in \mathbb{R}^n$, $r > 0$ a real #
$\text{Ball}(x, r) := \{y \in \mathbb{R}^n \mid \text{dist}(x, y) < r\}$

this is called the open ball of radius r about x



ex. ($n=2$) let $x=(1,3)$ and $r=2$

doesn't include dashes, only inside

POSETS:

ex. there are some sets w/ extra structures allowing us to compare elements.

$\mathbb{Z}$ has $<$ (eg. $3 \leq 19$)

$A = \{1, \text{banana}, \text{pineapple}\}$, $P(A)$ has a subset (eg. $\{1\} \subset \{1, \text{pineapple}\}$)

⚠ $\{1\}$ and $\{\text{banana}\}$ are incomparable

rmk: $\mathbb{Z}$ w/ $\leq$ is "totally ordered" (any 2 elements can be compared)
so $P(A)$ is partially ordered.

is there an order on $\mathbb{R}^2$? yes, many
ex. given (x_1, x_2) and (x_1', x_2')
$(x_1, x_2) \leq (x_1', x_2')$ if (1) $x_1 \leq x_1'$ or (2) $x_1 = x_1'$ and $x_2 \leq x_2'$
aka lexicographic order of $\mathbb{R}^2$

informal defn: fix a set P. A partial order/relation on P is a rule $\leq$ for comparing two elements of P satisfying:

(1) reflexive: $\forall x \in P, x \leq x$
(2) transitive: $\forall x, y, z \in P$
if $x \leq y$ and $y \leq z$, then $x \leq z$
(3) antisymmetry: if $x \leq y$ and $y \leq x$, then $x = y$

(rule ?: we need a way to decide if $(x,y) \in P \times P$ satifies the "rule.")

ex. $P(A)$:
(1) $S \in P(A)$, $S \subset A$
(2) $S, T, P \subset A$, if $S \subset T$ & $T \subset R$ then $S \subset R$
(3) if $S \subset R$ and $R \subset S$, then $S = R$

$\therefore$ $(P(A), \subset)$ is an example of a partial order
↳ operation

notation: let P be a set $\leq$ a partial order P. we say that the pair $(P, \leq)$ is a partially ordered set aka poset

"$\leq$" can be implicit and say "P is a poset", the $\leq$ is implied defn: given sets S, T. $S \times T$ is the set of ordered pairs (s,t) s.t. $s \in S$ & $t \in T$

order matters! $S \times T \neq T \times S$

"rule" is a subset $R \subset P \times P$ s.t. a subset is called a relation

ex. $P = \mathcal{P}(\{a,b\})$

$P \times P =$

(ϕ, ϕ)	$(\phi, \{a\})$	$(\phi, \{b\})$	$(\phi, \{a,b\})$
$(\{a\}, \phi)$	$(\{a\}, \{a\})$	$(\{a\}, \{b\})$	$(\{a\}, \{a,b\})$
$(\{b\}, \phi)$	$(\{b\}, \{a\})$	$(\{b\}, \{b\})$	$(\{b\}, \{a,b\})$
$(\{a,b\}, \phi)$	$(\{a,b\}, \{a\})$	$(\{a,b\}, \{b\})$	$(\{a,b\}, \{a,b\})$

every element is the first set is in the second

circle every pair (x,y) s.t. $x \leq y$

$S \subset T$

defn: fix a set P a partial order/relation on P is a subset $R \subset P \times P$ satisfying:

(1) $\forall x \in P$, $(x,x) \in R$

(2) $\forall x,y,z \in P$. if $(x,y) \in R$ and $(y,z) \in R$ then $(x,z) \in R$

(3) if $(x,y) \in R$ and $(y,x) \in R$, then $x = y$

$x \leq y$ to mean $(x,y) \in R$ and write $(P, \leq)$ for a poset.