

## Practice: Curve-Sketching & L'Hôpital's Rule

### Exercise 1: Evaluate

a)  $\lim_{x \rightarrow 1} \frac{\ln x}{x-1}$

e)  $\lim_{x \rightarrow \pi^-} \frac{\sin x}{1 - \cos x}$

i)  $\lim_{x \rightarrow \infty} (x - \ln x)$

b)  $\lim_{x \rightarrow \infty} \frac{e^x}{x^2}$

f)  $\lim_{x \rightarrow 0} \frac{6^x - 2^x}{x}$

j)  $\lim_{x \rightarrow \infty} e^{-x} \ln x$

c)  $\lim_{x \rightarrow 0} \frac{\sin x}{e^x}$

g)  $\lim_{x \rightarrow \infty} x^2 e^x$

k)  $\lim_{x \rightarrow 0} \frac{e^{4x} - 1}{\cos x}$

d)  $\lim_{x \rightarrow \infty} \frac{\ln x}{\sqrt[3]{x}}$

h)  $\lim_{x \rightarrow -\infty} x^2 e^x$

l)  $\lim_{x \rightarrow 0^+} x^x$  (Hint:  $x = e^{\ln x}$ )

### Exercise 2: Sketch the graph of a function $y=f(x)$ satisfying the following properties:

- i. The domain of  $f(x)$  is  $[-5, 6)$ .
- ii. The range of  $f(x)$  is  $(-\infty, 5]$ .
- iii.  $f'(0)=0$  and  $f'(x)<0$  for  $1<x<6$ .
- iv.  $f(x)$  has a vertical asymptote at  $x=6$ .
- v.  $f(x)$  is continuous on the interval  $[-5, 0]$  but not differentiable at  $x=-3$ .
- vi.  $f'(x)=2$  for  $-5<x<-3$ .
- vii.  $\lim_{x \rightarrow 1} f(x)$  does not exist.