Practice: Asymptotes & Curve-Sketching
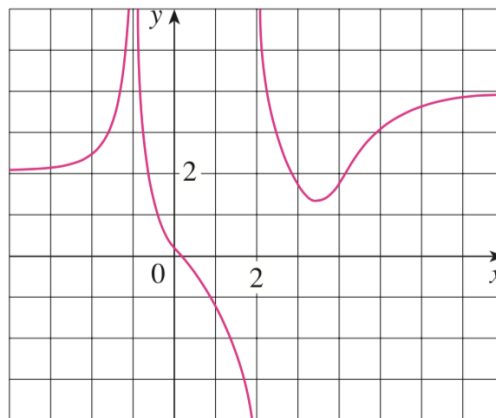
Exercise 1: Evaluate

a) $\lim\limits_{x\to\infty} \dfrac{3x^2-x-2}{5x^2+4x+1}$

b) $\lim\limits_{x\to\infty} \dfrac{x^2+x}{3-x}$

c) $\lim\limits_{x\to\infty} \left(\sqrt{x^2+1}-x\right)$

Exercise 2: Find the infinite limits, limits at infinity, and asymptotes for the function f whose graph is shown in the figure.

Exercise 3: Sketch the graph of $y = f(x)$ satisfying the following properties:

i.   The domain of f is $(-\infty, -2) \cup (-2, \infty)$ and f is continuous on its domain.

ii.   The intercepts of f are given by $f(-5) = 0, f(x) = 0, f(-1) = 0$ and $f(0) = -3$.

iii. $\lim\limits_{x\to\infty} f(x) = 2$ and $\lim\limits_{x\to-2} f(x) = \infty$

iv. $f'(x) > 0$ on $(-\infty, -2) \cup (0, \infty)$ and $f'(x) < 0$ on $(-2,0)$

v. $f''(x) > 0$ on $(-5, -2) \cup (-2,3)$ and $f''(x) < 0$ on $(-\infty, -5) \cup (3, \infty)$

Exercise 4: Without using a graphing calculator, sketch the curves:

a) $y = \dfrac{2x^2}{x^2-1}$

b) $y = \dfrac{1}{e^x - 5}$

Exercise 5: Find the horizontal and vertical asymptotes of the graph of the function

$$f(x) = \frac{\sqrt{2x^2 + 1}}{3x - 5}$$