Lab worksheet for Thursday, April 15, 2021

## Practice: IVT \& Limits equaling $\infty$

## Exercise 1:

a) Show that there is a root of the equation $4 x^{3}-6 x^{2}+3 x-2=0$ between 1 and 2 .
b) Can you find a better approximation for that root?

## Exercise 2:

a) Show that there is a solution to the equation $\cos x=x$ on the interval $(0,1)$.
b) If $f(x)=x^{2}+10 \sin x$, show that there is a number c such that $\mathrm{f}(\mathrm{x})=1000$.

Exercise 3: Suppose $f$ is continuous on [1,5] and the only solutions of the equation $f(x)=6$ are $x=1$ and $x=4$. If $f(2)=8$, explain why $f(3)>6$.

Exercise 4: Compute the following limit if it exists. If it does not exist, explain why.
a) $\lim _{x \rightarrow 3} \frac{2 x}{x-3}$
b) $\lim _{x \rightarrow-3} \frac{x+2}{x+3}$
c) $\lim _{x \rightarrow 0} \frac{2-x}{x^{2}}$
d) $\lim _{x \rightarrow 1} \frac{2-x}{(x-1)^{2}}$
e) $\lim _{x \rightarrow 2^{-}} \frac{x^{2}-2 x}{x^{2}-4 x+4}$

## Exercise 5:

a) The graph of the function $f(x)=\tan x$ is given below. What can we say about the limits $\lim _{x \rightarrow \frac{\pi}{}^{+}} \tan x, \lim _{x \rightarrow \frac{\pi^{-}}{2}} \tan x$, and $\lim _{x \rightarrow \frac{\pi}{2}} \tan x$ ?

b) Given that $\tan x=\frac{\sin x}{\cos x}$. Justify your answer intuitively by looking at what happens to $\sin x$ and $\cos x$ when $x$ approaches $\frac{\pi}{2}$ from the left and from the right.

