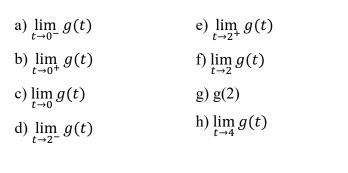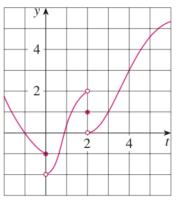**Lab worksheet for Thursday, 1 April 2021**

Practice: Limits and one-sided limits, informally

Exercise 1: For the function g whose graph is given, state the value of each quantity, if it exists/is defined. If it doest not exist/is not defined, explain why.

a) $\lim\limits_{t \to 0^-} g(t)$

b) $\lim\limits_{t \to 0^+} g(t)$

c) $\lim\limits_{t \to 0} g(t)$

d) $\lim\limits_{t \to 2^-} g(t)$

e) $\lim\limits_{t \to 2^+} g(t)$

f) $\lim\limits_{t \to 2} g(t)$

g) g(2)

h) $\lim\limits_{t \to 4} g(t)$

Exercise 2: Is the function g (in exercise 1) continuous at x = 0, x=2, x=4? Explain your answer.

Exercise 3: For the function h whose graph is given, state the value of each quantity, if it exists/is defined. If it doest not exist/is not defined, explain why.
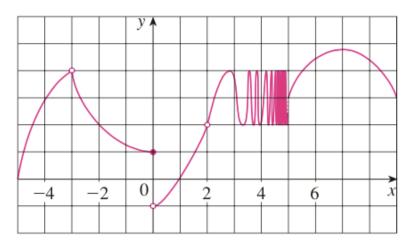
a) $\lim\limits_{x \to -3^-} h(x)$

b) $\lim\limits_{x \to -3^+} h(x)$

c) $\lim\limits_{x \to -3} h(x)$

d) $h(-3)$

e) $\lim\limits_{x \to 0^-} h(x)$

f) $\lim\limits_{x \to 0^+} h(x)$

g) $\lim\limits_{x \to 0} h(x)$

h) $h(0)$

i) $\lim\limits_{x \to 2} h(x)$

j) h(2)

k) $\lim\limits_{x \to 5^+} h(x)$

l) $\lim\limits_{x \to 5^-} h(x)$

Exercise 4: Sketch the graph of the following function and use it to determine the values of a for which $\lim\limits_{x \to a} f(x)$ exists:

$$f(x) = \begin{cases} 2 - x & \text{if } x < -1 \\ x & \text{if } -1 \leq x < 1 \\ (x-1)^2 & \text{if } x \geq 1 \end{cases}$$

Exercise 5: Sketch the graph of an example of a function f that satisfies all of the given conditions:

1. $\lim\limits_{x \to 3^+} f(x) = 4$

2. $\lim\limits_{x \to 3^-} f(x) = 2$

3. $\lim\limits_{x \to -2} h(x) = 2$

4. $f(3) = 3$

5. $f(-2) = 1$

Exercise 6: The sign function, denoted by sgn, is defined by

$$sgn(x) = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$$

a) Sketch the graph of this function.

b) Find each of the following or explain why it does not exist.

i) $\lim\limits_{x \to 0^+} sgn(x)$ iii) $\lim\limits_{x \to 0} sgn(x)$

ii) $\lim\limits_{x \to 0^-} sgn(x)$ iv) $\lim\limits_{x \to 0} |sgn(x)|$

Exercise 7: Consider the Dirichlet function:

$$f(x) = \begin{cases} 1 & \text{if x is rational} \\ 0 & \text{if x is irrational} \end{cases}$$

Find $\lim\limits_{x \to 0^+} f(x)$, $\lim\limits_{x \to 0^-} f(x)$ if they exist, if not, explain why.