# Lecture 23

# Average values

**Definition 23.0.1.** Let $f$ be a function and choose two real numbers $a$ and $b$ such that $a < b$. Then *the average value* of $f$ over the interval $[a, b]$ is defined to be

$$\frac{\int_a^b f(x)\,dx}{b-a}.$$

Of course, the above is equal to

$$\frac{1}{b-a}\int_a^b f(x)\,dx.$$

**Example 23.0.2.** A bird travels. The velocity at time $t$ is given by the function $v(t) = t^2$ kilometers per second. What is the average velocity with which the bird was traveling between $t = 0$ and $t = 3$?

Solution. We must compute

$$\begin{aligned}
\frac{1}{b-a}\int_a^b v(t)\,dt &= \frac{1}{3-0}\int_0^3 t^2\,dt && (23.0.1)\\
&= \frac{1}{3}\left(\frac{1}{3}t^3\Big|_0^3\right) && (23.0.2)\\
&= \frac{1}{3}\left(\frac{1}{3}\left((3)^3-(0)^3\right)\right) && (23.0.3)\\
&= 3. && (23.0.4)
\end{aligned}$$

The average speed of the bird is 3 kilometers per second.

**Remark 23.0.3.** The above example should give you an idea of why this really is an average value. Remember that when $v$ is velocity, $\int_a^b v\,dt$ computes the distance traveled. So to divide this value by $b - a$ is to divide total distance traveled by the time elapsed. As you know, this measures how quickly, on average, one is traveling over that time. In other words, this does indeed compute average velocity.

## 23.1 Practice day!

The rest of today will just be practicing integrals.

Compute the following indefinite integrals and (where bounds are indicated) definite integrals. They are taken from Guichard's textbook.

**Exercise 23.1.1.** $\int (1-t)^9 \, dt$

**Exercise 23.1.2.** $\int (x^2+1)^2 \, dx$

**Exercise 23.1.3.** $\int x(x^2+1)^{100} \, dx$

**Exercise 23.1.4.** $\int \frac{1}{(1-5t)^{1/3}} \, dt$

**Exercise 23.1.5.** $\int \sin^3 x \cos x \, dx$

**Exercise 23.1.6.** $\int x\sqrt{100-x^2} \, dx$

**Exercise 23.1.7.** $\int \frac{x^2}{\sqrt{1-x^3}} \, dx$

**Exercise 23.1.8.** $\int \cos(\pi t)\cos(\sin(\pi t)) \, dt$

**Exercise 23.1.9.** $\int \frac{\sin x}{\cos^3 x} \, dx$

**Exercise 23.1.10.** $\int \tan x \, dx$

**Exercise 23.1.11.** $\int_0^\pi \sin^5(3x)\cos(3x) \, dx$