Lab: Word problems for max/min

12.1

A ballerino leaps. The height of his center of mass at time t is given by

$$h(t) = 6t - 9.8t^2.$$

At what time is his center of mass highest?

12.2

You may use a calculator for this exercise. Your friend Yadira is running an online banking business. For whatever reason, she models the profit P of her company as a function of the cost x as follows:

$$P(x) = \frac{2}{\sqrt{2}} \sin(x) + x.$$

All units are in bitcoins. (So when her company spends x bitcoins in cost, they make $P(x)$ bitcoins in profit.)[2] Yadira's company budget is 4.2 bitcoins, meaning she cannot spend more than 4.2 bitcoins.

   According to this model, how much should Yadira's company budget (i.e., how much should cost be) to maximize her company's profit?

12.3

The population of buffalo at Shimizu Ranch is modeled by the function

$$B(t) = (3t - 5)e^t + 8$$

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[2]By the way, it is (very!) possible to buy fractions of bitcoins; people routinely buy 0.000000001 bitcoins, for example.

where t is in units of years from now, and the population of buffalo, $B(t)$, is in hundreds of buffalo. (For example, at $t = 0$—i.e., right now—there are 300 buffalo. At $t = 1/3$—i.e., 4 months from now—there will be approximately $(8 - 2e^{1/3}) \cdot 100$ buffalo.)

You plan on visiting Shimizu Ranch at some point within the next 2 years and four months. (That is, sometime within the next $2\frac{1}{3}$ years.)

(a) Your friend Venus loves buffalo, and wants to see as many buffalo as possible. At what time during the next $2\frac{1}{3}$ years should you visit the ranch to go when there are the most buffalo?

(b) But your brother, MJ, is terrified of buffalo, and wants to go when there are as few buffalo as possible. At what time during the next $2\frac{1}{3}$ years should you visit the ranch to go when there are the fewest buffalo?

12.4

A statue is built to have the shape of the graph

$$y(x) = |\ln(x^2 - 2x + 1.5)|$$

over the interval $[0, 2]$.

(a) At what point(s) is the statue lowest to the ground?

(b) At what point(s) is the statue tallest?

You will need to do more than just find the critical points. (Hint: There are points where the derivative of y is not defined. What can you say about those points?)

12.5

Mboyo has built a wonky ferris wheel. Your height at time t—where t is measured in hours, and you get onto the ferris wheel a time $t = 0$, is measured by

$$h(t) = 10e^{\sin(2\pi t)}$$

where h is in meters. (So for example, when you get on initially, your height is 10 meters off the ground.)

Mboyo is so proud of his ferris wheel that he makes you ride for seven hours.

(a) You want your camera to automatically take photos from the highest points of the ride. For what time, or for what times, should you program your camera timer so that your camera automatically takes photos when you are highest on the ride?

(b) Your young son is afraid of heights, and he says he only wants to open his eyes when the ride is at its lowest points. At what time, or at what times, should you tell him to open his eyes?