Lab worksheet for Thursday, 4 Feb 2021

Practice: Derivatives of Logarithms

1. Compute the following without calculators

a) $\ln(e^3)$.

b) $e^{\ln(3)}$.

c) $e^{\ln 2 + \ln 3}$.

d) $e^{\ln 6 - \ln 2}$.

2. Find the derivatives of the following fuctions:

a) $f(x) = \ln(x^3 + 1)$.

b) $g(x) = 2\ln(\cos x + x^2 - 2x)$.

c) $h(x) = \ln(e^x + 2x^2)$.

3. Find the derivatives of the following fuctions:

a) $f(x) = \log_2 x$.

b) $h(x) = 2\log_3(6x)$.

b) $g(x) = 3\log_5(3x^2+1)$.

4. A Cessna plane takes off from an airport at sea level and its altitude (in feet) at time t (in minutes) is given by $h = 2000 \ln (t + 1)$. Find the rate of climb at time t = 3 min.

5. At what point on the curve $y = 1 + 2e^x – 3x$ is the tangent line parallel

to the line $3x – y = 5$? Illustrate by graphing the curve and both lines.

6. Find the derivatives of the following fuctions:

a) $y = x^{-\frac{2}{5}}$.

b) $A(s) = -\frac{12}{s^5}$.

c) $G(x) = \sqrt[4]{1 + 2x + x^3}$.

7. Find the derivatives of the following fuctions:

a) $y = \ln(2x^3 – x)^2$.

b) $y = \ln(\cos (x^2))$.

KEY

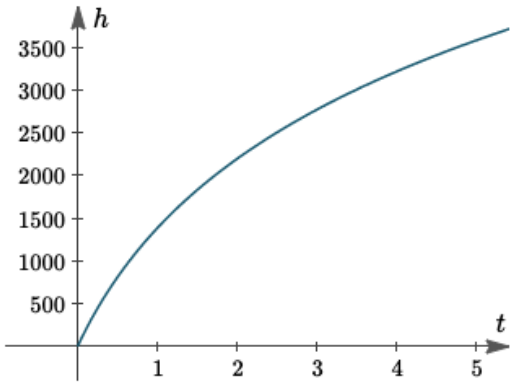
1. a) 3 b) 3 c) 6 d) 3

2. a) $\frac{3x^2}{x^3+1}$ b) $\frac{2(-sinx+2x-2)}{cosx+x^2-2x}$ c) $\frac{e^x+4x}{e^x+2x^2}$

3. a) $\frac{1}{x.ln2}$ b) $\frac{2}{x.ln3}$ c) $\frac{18x}{(3x^2+1).ln5}$

4.

The graph of $h = 2000 \ln(t+1)$ shows that it is a realistic model for the climb performance of a light aircraft. At low altitudes, where the air is more dense, the rate of climb is good, but as you go higher, the rate decreases.



The graph of $h = 2000 \ln(t+1)$.

To find the rate of climb (vertical velocity), we need to find the first derivative:

$$\frac{d}{dt} 2000 \ln(t+1) = \frac{2000}{t+1}$$

At $t = 3$, we have $v = 2000/4 = 500$ feet/min.

So the required rate of climb is $500'/\text{min}$, which is quite realistic.

5. $y' = 2e^x - 3$.

$y' = 3$ when $2e^x = 6$, $e^x = 3$, $x = \ln 3$. Then $y = 1 + 6 - 3\ln 3$.

6. a) $\frac{-2}{5}.x^{\frac{-7}{5}} = \frac{-2}{5\sqrt[5]{x^7}}$ b) $\frac{60}{s^4}$ c) $\frac{3x^2+2x}{4\sqrt[4]{1+2x+x^3}^3}$

7. a) $\frac{2(6x^2-1)}{2x^3-x}$ b) $\frac{-2x.sin(x^2)}{cos(x^2)} = -2x.\tan(x^2)$