

Lab: Computing derivatives by hand

Lab Problem 1 (Lines). Let $f(x) = 3x + 2$.

- (a) What does the graph of f look like?
- (b) Based on your previous answer, and based on the equation of f , what do you expect the slope of *any* secant line to be?
- (c) Let $x = 2$. What is the value of the fraction

$$\frac{f(x+h) - f(x)}{h}$$

for different values of h ? (Does it even depend on h)? What value does it take as h approaches zero?

- (d) Do the same problem as above, but with $x = 3$.
- (e) For this function— $f(x) = 3x + 2$ —does the derivative of f at x depend on which value of x you choose?

Lab Problem 2 (Parabolas). Now let $g(x) = 5x^2$.

- (a) Let $x = 2$. Does the fraction

$$\frac{g(x+h) - g(x)}{h}$$

simplify when h does not equal zero? What value does it take as h approaches zero? In other words, what is the derivative of g at 2?

- (b) Do the same problem as above, but with $x = 3$.

- (c) Do the same problem as above, but with $x = 4$.
- (d) Do you notice a relationship between the derivative of g at x , and x ? Put another way, is there a formula that takes x as an input, and outputs the derivative of g at x ?

Lab Problem 3 (Parabolas again). Now let $q(x) = 5x^2 + 9$. (How is this function different from the previous function?)

- (a) What is the derivative of q at $x = 2$?
- (b) Do the same problem as above, but with $x = 3$.
- (c) Do the same problem as above, but with $x = 4$.
- (d) Do you notice a relationship between the derivative of q at x , and x ? Put another way, is there a formula that takes x as an input, and outputs the derivative of q at x ?
- (e) How does the derivative of q relate to the derivative of g ? How are q and g themselves related?
- (f) What if we had added 5, or 2, or π , or one million to g ?

Lab Problem 4 (Adding functions). Now let $p(x) = 5x^2 + 3x + 2$. (How does this compare to the first two functions you studied— f and g ?)

- (a) What is the derivative of p at $x = 2$?
- (b) Do the same problem as above, but with $x = 3$.
- (c) Do the same problem as above, but with $x = 4$.
- (d) Is there a formula that takes x as an input, and outputs the derivative of p at x ?
- (e) How does the derivative of p relate to the derivatives of g and q ?

Lab Problem 5 (Cubics). Let $r(x) = x^3$.

- (a) What is the derivative of r at $x = 2$?
- (b) Do the same problem as above, but with $x = 3$.
- (c) Do the same problem as above, but with $x = 4$.
- (d) Is there a formula that takes x as an input, and outputs the derivative of r at x ?