Extra Credit Assignment 5

Due Friday, March 12, 11:59 PM

Respond to one of the following prompts. Your choice.

1. We've taken the power rule for granted. That is, that
$$(x^n)' = nx^{n-1}.$$
Tell me *why* the power rule is true.

2. Here is a kind of "related rates" problem, but quite different. Usually in related rates problems, we know how a function depends on other factors (often via a formula), and we can deduce the rate of change of the function in terms of those factors.

   But here is a different kind of problem: Suppose you know that the rate of change of something is *proportional* to the quantity of that something. (And this proportion does not change with time.) In other words,
$$f'(t) = Cf(t)$$
for some constant C. (Somebody might demand that C equal 5, or 0, or -1, or $1/\pi$; each such choice of C results in a different equation.)

   What kind of function could $f(t)$ be? Can you write down all functions f that satisfy the above equation? Can you write down an f that satisfies the above equation for $C = 5$? That is, can you write down an f so that
$$f'(t) = 5f(t)?$$
What if you change C? And can you write down *all* such functions? How can you convince me that you've written down all of them?

3. Is there such a thing as a "smallest positive real number?" Explore. Why or why not?

4. Is there such a thing as a number called ∞? If so, what properties would/should/does it satisfy? Can you add with ∞? Subtract with it? Multiply, divide by it? Exponentiate it, or use it as a power? Can you take sin or cos of tan or arctan of it? Explore. Why or why not?