

Extra Credit Assignment 1: Rational numbers

Due Friday, January 22, 11:59 PM

An *integer* is a positive or negative whole number. For example, -5, 2, 0, and 9999 are all integers.

A *rational number* is a number that can be written as a fraction of two integers. For example,

$$\frac{-3}{2}, \quad \frac{1}{1}, \quad \frac{3}{1}, \quad \frac{14}{7}, \quad \frac{1}{3},$$

are all rational numbers. (Note that the middle three also happen to be equal to integers! So integers are a special kind of rational number.)

You may have heard, or learned, at some point in your life that *if you write out the decimal expansion of a rational number, the expansion will eventually begin to repeat some string of digits*. For example,

$$\frac{1}{3} = 0.3333333 \dots$$

repeats the number 3 over and over,

$$\frac{789}{1000} = 0.78900000000 \dots$$

eventually repeats the number 0 over and over, while

$$\frac{523242}{7000} = 74.748857142857142857142857 \dots$$

eventually repeats the sequence “142857” over and over.

Prompt. Why is the italicized statement above true?

Also, in the “opposite” direction: Is it true that if a decimal number eventually repeats some string of digits over and over, it must be a rational number?