1.

For each function f below, find an **antiderivative** of f.

(i) $f(x) = \sin(x) + \cos(x)$
(ii) $f(x) = e^x + x$
(iii) $f(x) = x^5 + 4x^3$
(iv) $f(x) = \frac{1}{1+x^2}$
(v) $f(x) = \frac{1}{x}$
(vi) $f(x) = \sec^2(x)$

2.

Compute each limit below.

(i) $\displaystyle\lim_{x\to 0^-} \frac{1}{x}$

(ii) $\displaystyle\lim_{x\to\infty} \sin(x)$

(iii) $\displaystyle\lim_{x\to-\infty} \frac{x-1}{x+1}$

(iv) $\displaystyle\lim_{x\to-1^-} \frac{x-1}{x+1}$

3.

(i) Sketch (or explain why it is impossible to sketch) the graph of a function f such that

- $\lim_{x\to\infty} f(x) = 3$
- $\lim_{x\to-\infty} f(x) = -3$
- $\lim_{x\to 2^-} f(x) = -\infty$
- $\lim_{x\to 2^+} f(x) = \infty$
- $f''(x) > 0$ for all $x < 2$.
- $f''(x) > 0$ for all $x > 2$.

(ii) Sketch (or explain why it is impossible to sketch) the graph of a function f such that

- $\lim_{x\to\infty} f(x) = 3$
- $\lim_{x\to-\infty} f(x) = -3$
- $\lim_{x\to 2^-} f(x) = -\infty$
- $\lim_{x\to 2^+} f(x) = \infty$
- $f''(x) < 0$ for all $x < 2$.
- $f''(x) > 0$ for all $x > 2$.

4.

(a) Find the derivative with respect to x of the function given below.

$$f(x) = (3x - 1)^4 \sin(8x)$$

(b) Identify any differentiation rules that you used to solve this problem.
(c) Explain why you chose to use each of the rules you identified in part (b).

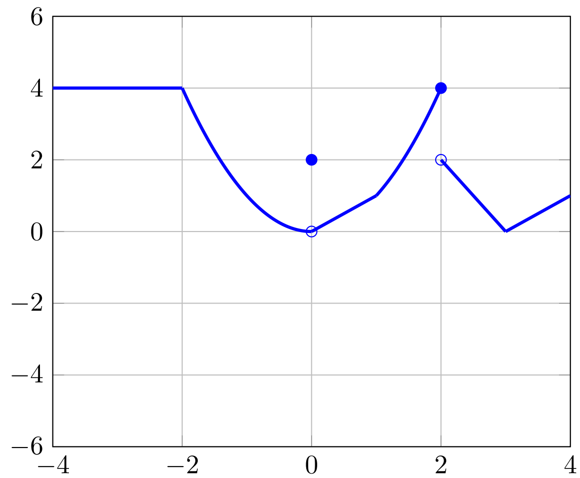
5.

Let $f(x) = x^2 – 6x + 10$ on $[-1, 4]$.

(a) Find every critical point.
(b) Find the absolute maximum.
(c) Find the absolute minimum.

6.

Below is a graph of a function f. State whether the following statements are true or false.



(a) $f'(-3) = 0$.

(b) $\lim_{x\to 0} f(x) = 0$.

(c) f is continuous at $x = 0$.

(d) $\lim_{x\to 2^+} f(x) = 2$

(e) $\lim_{x\to 2^-} f(x) = 2$

(f) f has a derivative at $x = -2$

(g) f is continuous at $x = -2$.

(h) $f(0) = 2$.

7.

Complete the following definitions.

(a) Let f be a function. The *derivative of f at x* is:

(b) Let f be a function. We say that f is *continuous at a* if:

(c) Let f be a function. The *integral* of f along the interval $[a, b]$, written as $\int_a^b f(x)\,dx$, is defined to be:

8.

Compute the following integral:

$$\int_1^2 \frac{4x^2}{3x^3 - 2}\,dx$$

9.

Find the area of the region enclosed by the curves $y = 3x - x^2$ and $y = -4$.

10.

A gardener wants to enclose a rectangular garden with area 2000 square feet. They want to use fencing on the north, east, and west sides of the rectangular garden that costs \$3 per foot. They want to use a fancier fence on the south side of the garden that costs \$8 per foot.

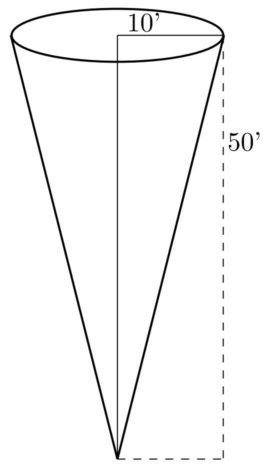
(a) What dimensions should they make the garden in order to minimize the cost?

(b) Explain why the critical point you have chosen gives the minimum possible cost.

(c) What would be the minimum cost for a garden with these dimensions?

11.

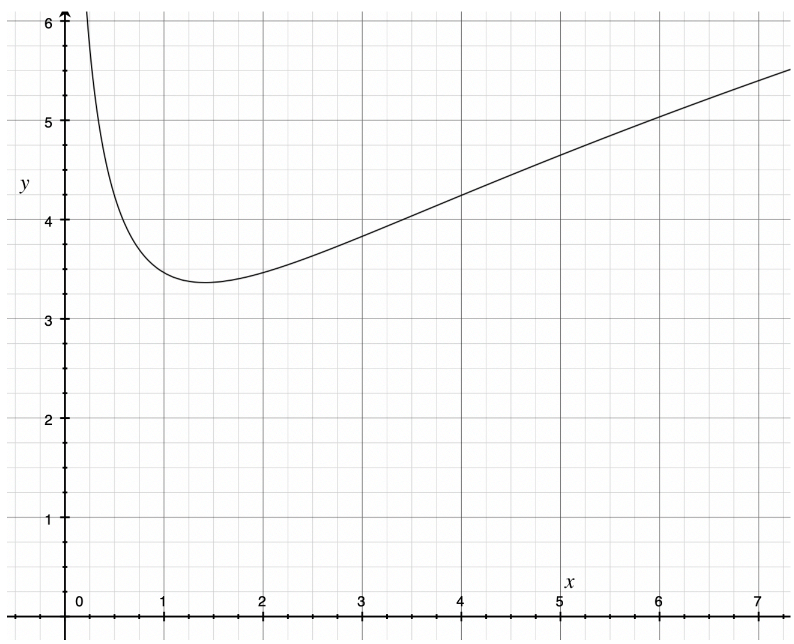
A conical container with dimensions shown on the figure below is initially full of water. A valve is opened and water flows out at a rate of 10 cubic feet per minute. How fast is the depth of the water changing when the depth is 20 feet?



(The formula for the volume of a cone: $V = \frac{1}{3}\pi r^2 h$.)

12.

A part of the solution set to the equation $4x^2+8 = xy^2$ is graphed below.



(a) By looking at the graph, how do you expect the slope of the tangent line at the point $(1, \sqrt{12})$ to compare to the slope of the tangent line at the point $(2, \sqrt{12})$?
(b) Find $\frac{dy}{dx}$ by using implicit differentiation.
(c) Find the exact slopes of the tangent lines at the points $(1, \sqrt{12})$ and $(2, \sqrt{12})$.
(d) By looking at the graph, between what two consecutive integer x-values do you expect to find that the tangent line is horizontal?
(e) Find the exact x-value where the tangent line is horizontal.
[Hint: use the substitution $y^2 = \frac{4x^2+8}{x}$ to help you solve.]

13.

Hiro has modeled the height of the tide at Galveston Bay by the function

$$h(t) = 0.6 + 0.7\cos(\frac{\pi}{12}t)$$

where the height $h(t)$ is measured in feet above sea level, and t is measured in hours (from midnight, so $t = 0$ is midnight, while $t = 6$ is 6 AM).

In the six-hour period from midnight to 6 AM, on average, how high was the tide at Galveston Bay?

14.

(a) State the fundamental theorem of calculus.
(b) State the mean value theorem.
(c) State the intermediate value theorem.

15.

Consider the statement: "**If** $\lim_{x\to a} f(x) = L$, **then** for all $\epsilon > 0$, there exists a $\delta > 0$ such that $0 < |x - \delta| < \delta \implies |f(x) - L| < \epsilon$."

State the contrapositive of the above statement.

16.

State L'Hopital's rule. Make sure to be explicit about *when* we can use L'Hopital's rule.

17.

Let $f(x) = \sin(x)$.

Find a degree 3 polynomial $T(x)$ so that

- $T(0) = f(0)$,
- $T'(0) = f'(0)$,
- $T''(0) = f''(0)$, and
- $T^{(3)}(0) = f^{(3)}(0)$.

18.

Compute the indefinite integral

$$\int \ln x \, dx$$

showing all work. (This is hard.)