

## 0.1 November 16th Lab: More long-term review for the final

**Exercise 0.1.1.** Write the 5th degree Taylor polynomial for  $f(x) = \sin(x)$ , centered at  $a = 0$ .

**Exercise 0.1.2.** Compute the following integrals.

(a)  $\int_0^4 x^3 dx$

(b)  $\int_0^\pi \sin(x)e^{\cos(x)} dx$

(c)  $\int_1^3 \frac{2+2x^2}{x+\frac{1}{3}x^3} dx$

**Exercise 0.1.3.** We have 100 feet-squared of cardboard to build a box with a square base and no top. What dimensions of the box will maximize the volume?

**Exercise 0.1.4.** (a) State the definition of the derivative of a function  $f$  at a point  $x$ .

(b) Using this definition, compute the derivative of  $f(x) = |x|$  when  $x > 0$ .

(c) Using this definition, compute the derivative of  $f(x) = |x|$  when  $x < 0$ .

(d) Using this definition, explain why  $f(x) = |x|$  has no derivative where  $x = 0$ .

**Exercise 0.1.5.** It is known that the gravitational force that a planet exerts on a spaceship is given by

$$\frac{km}{r^2}$$

where  $m$  is the mass of the spaceship,  $k$  is some constant, and  $r$  is the distance of the spaceship from the center of the planet.

- (a) How much work must be done to move a spaceship from 10,000 kilometers away from the center of the planet to 100,000 kilometers away? (Your answer should contain  $k$  and  $m$  in its answer.)
- (b) How much work must be done to move a spaceship from 10,000 kilometers away from the center of the planet to 1,000,000 kilometers away? (Your answer should contain  $k$  and  $m$  in its answer.)
- (c) How much work must be done to move a spaceship from 10,000 kilometers away from the center of the planet to 10,000,000 kilometers away? (Your answer should contain  $k$  and  $m$  in its answer.)
- (d) \* How much work must be done to move a spaceship from 10,000 kilometers away from the center of the planet to a point “infinitely far away”? (Your answer should contain  $k$  and  $m$  in its answer.)