Lecture 28

More more more practice!

28.1 Exam format

Hiro will send out an e-mail with instructions for the exam, just as he did for last exam.

As before, there will be true-false questions, multiple choice questions, some computational questions (compute the interval), some definitions (you should know the definitions of integral, integrand, and average value), and some word problems (you should know that the integral of a rate of change is the net change—e.g., integral of velocity gives you net change in position). You should also know what the Fundamental Theorem of Calculus is.

The exam will be in the *class Zoom* (NOT the lab Zoom) at 8 AM Thursday.

28.2 Word problems

Exercise 28.2.1. Suppose $F(t) = 7t^2 - 9$. Compute

$$\int_3^4 F'(t)\,dt.$$

Exercise 28.2.2. The nucleus of the hydrogen atom exerts a force of

$$\frac{(9 \times 10^9) \times (1.6 \times 10^{-19})^2}{r^2}$$

(in Newtons) on an electron r meters away from the nucleus.

1. It is common to pretend that a typical electron is 5.3×10^{-11} meters away from the nucleus. Pretending this, what is the force exerted by the nucleus on this electron? You do not need to simplify your answer.
2. How much work does it take to move an electron from 5.3×10^{-11} meters away, to 100 meters away from the nucleus?

   Set up the integral. You do not need to compute it.

Exercise 28.2.3. Compute the following indefinite integrals.

(a) $\int (3-2t)^8 \, dt$.

(b) $\int \tan(x) \, dx$.

(c) $\int \frac{x-1}{3x^2-6x} \, dx$

(d) $\int 7e^x \cos(e^x) \, dx$

(e) $\int \frac{1}{1+x^2} \, dx$

(f) $\int \frac{3}{1+x^2} \, dx$

(g) $\int x^3 + 2x - 9 \, dx$

Exercise 28.2.4. Write out all the terms in the following summations. For example,

$$\sum_{b=2}^{5} 3b = 3 \times 2 + 3 \times 3 + 3 \times 4 + 3 \times 5.$$

For this problem, you do not need to compute what the sum is.

(a)

$$\sum_{i=3}^{5} \cos(i\pi)$$

(b)

$$\sum_{a=1}^{4} a^2$$

(c)

$$\sum_{n=2}^{6} \frac{1+n}{n^2}$$

(d)

$$\sum_{j=0}^{3} (-1)^j$$

28.3 Challenge problems

Exercise 28.3.1. Using Σ summation notation, write out what the degree 5 Taylor polynomial for a function f is, centered at a point a. As a hint, you can write $f^{(n)}(a)$ to mean the nth derivative of f, evaluated at $x = a$.

Exercise 28.3.2. Using Riemann sums, can you explain to me why the statements

(i) You can make $\ln(x)$ as big as you want so long as you make x bigger, and

(ii) You can make the sum $\sum_{n=1}^{L} = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{L}$ as big as you want so long as you make L bigger

are either both true, or both false?