

Lecture 27

More more practice!

27.1 Word problems

Exercise 27.1.1. $V(t)$ measures the amount of water (in liters) in a water tank at time t (in hours from midnight last night).

- (a) What units does $V'(t)$ have?
- (b) Suppose $V(t) = 100 - 2^t$. On average, how much water is in the water tank between $t = 2$ and $t = 3$?

Exercise 27.1.2. $F(t)$ measures the amount of water (in liters per hour) entering or leaving a water tank at time t (in hours from midnight last night). If $F(t)$ is positive, it means the amount of water in the water tank is increasing, while if $F(t)$ is negative, the amount of water is decreasing.

- (a) What units does $F'(t)$ have?
From hereon, suppose $F(t) = -10 + t^2$.
- (b) On average, how quickly is the volume of water in the tank changing between $t = 1$ and $t = 3$? Make sure to specify if, on average, the amount of water in the tank is decreasing or increasing.
- (c) Compute $\int_0^4 F(t) dt$.
- (d) Give a physical interpretation to the answer from your previous question.
- (e) Are you able to tell me how much water is in the water tank at $t = 4$?

Exercise 27.1.3. A gig worker is paid at a rate of $r(t) = 10 + \sin(\frac{1}{12\pi}t)$, where $r(t)$ is in dollars per hour, and t is in hours from midnight.

- (a) How much does the worker make if they work from 9 AM to 5 PM?
- (b) How does their 9 AM - 5 PM earnings compare to the 9 AM - 5 PM earnings of someone working at a flat rate of 10 dollars an hour?

(Warning: The function $r(t)$ used here was arbitrary, and does not in any way purport to realistically model the wages of particular gig workers.)

Exercise 27.1.4. (a) Give the definition of $\int_a^b f(t) dt$ that we have used in this class.

- (b) State the fundamental theorem of calculus, as we have learned it in this class.

Exercise 27.1.5. Planet X exerts a force of $\frac{100}{x^2}$ Newtons on a box x kilometers away from the center of planet X.

How much work does it take to move the box from 1,000 kilometers away to 10,000 kilometers away from Planet X?