Lecture 26

More practice!

26.1 Taking antiderivatives

Compute the following indefinite integrals. Many are taken from Guichard's textbook.

Exercise 26.1.1. $\int 25 - x^2\,dx$

Exercise 26.1.2. $\int (1-t)^9\,dt$

Exercise 26.1.3. $\int (x^2+1)^2\,dx$

Exercise 26.1.4. $\int x(x^2+1)^{100}\,dx$

Exercise 26.1.5. $\int \frac{1}{(1-5t)^{1/3}}\,dt$

Exercise 26.1.6. $\int \sin^3 x \cos x\,dx$

Exercise 26.1.7. $\int x\sqrt{100-x^2}\,dx$

Exercise 26.1.8. $\int \frac{x^2}{\sqrt{1-x^3}}\,dx$

Exercise 26.1.9. $\int \cos(\pi t)\cos(\sin(\pi t))\,dt$

Exercise 26.1.10. $\int \frac{\sin x}{\cos^3 x}\,dx$

Exercise 26.1.11. $\int \tan x\,dx$

Exercise 26.1.12. When computing the indefinite integral, we write

$$\int f(x)\,dx = F(x) + C$$

where $F(x)$ is an antiderivative of $f(x)$.

Tell me why we write "$+C$."

26.2 Computing definite integrals

Exercise 26.2.1. Evaluate

$$\int_1^4 \frac{3}{x^2}\,dx$$

Exercise 26.2.2. Evaluate

$$\int_0^\pi \sin^5(3x)\cos(3x)\,dx$$

Exercise 26.2.3. Evaluate

$$\int_1^{e^2} \frac{1}{x}\,dx$$

Exercise 26.2.4. Evaluate

$$\int_1^8 \frac{3x^2+2}{\sqrt{x}}\,dx$$

Exercise 26.2.5. Evaluate

$$\int_0^{2\pi} 8\cos(x)\,dx$$

Exercise 26.2.6. Evaluate

$$\int_1^8 2x+10\,dx$$

26.3 Areas between curves

Exercise 26.3.1. Find the area between the graphs of $x^2+2x-10$ and $4x-7$.

Exercise 26.3.2. Find the area between the three curves $y=x$ and $y=7x$ and $x=1$. (You may want to draw a picture.)

Exercise 26.3.3. Find the area between the graphs of x^3 and x^2.

26.4 Average values

Exercise 26.4.1. Find the average value of $f(x)=3+2x^2$ on the interval $[0,\sqrt{3}]$.

Exercise 26.4.2. An object attached to a (horizontally aligned) spring moves with velocity $v(t)=\sin(t)$.

(a) What is the average velocity of this object over the interval $[0, 2\pi]$?

(b) What is the average velocity of this object over the interval $[0, \pi]$?

Exercise 26.4.3. The number of new infections per day at the beginning of an outbreak can be modeled by the function $f(t) = e^t$, where t is in days and $f(t)$ is in units of (new) infections per day.

(a) At day 10, how many new infections are arising per day? (You can use a calculator if you want a decimal answer.)

(b) Between day 0 and day 10, on average, how many new infections have there been per day? (You can use a calculator if you want a decimal answer.)

26.5 Word problems

Exercise 26.5.1. A particle moves with velocity function $v(t) = -t^2 + 3x - 2$. Find the displacement (the signed distance between the starting and ending point) of the particle over the time interval $[-2, 3]$.

Exercise 26.5.2. An epidemiologist models that by day t of a pandemic, her county will have accumulated

$$\int_0^t 10e^{5x}\,dx$$

infections total. According to this, how many new infections per day will her county be seeing at the moment $t = 5$? (Hint: Fundamental Theorem.)

Exercise 26.5.3. It is known that (in the absence of air resistance) if you drop an object at time $t = 0$ seconds, the object's speed at time t is given by

$$v(t) = 32t$$

in feet per second. You drop a coin off a bridge, and you hear the coin hit the water after 10 seconds. Assuming no air resistance, how far above the water was the coin when you dropped it?

Exercise 26.5.4. Your friend is studying an ellipse given by the equation

$$(2x - 3)^2 + y^2 = 8^2.$$

Write an integral, or an expression involving an integral, that computes the area of the portion of this ellipse above the x-axis.

Exercise 26.5.5. Your friend is studying a circle given by the equation

$$(x-2)^2 + y^2 = 4$$

Write an integral, or an expression involving an integral, that computes the area of the portion of this circle above the x-axis.

26.6 Riemann sums

Exercise 26.6.1. (a) Write down, using Σ notation, a Riemann sum approximating the integral

$$\int_1^5 7x\,dx$$

using $n = 4$ rectangles and using either the lefthand or righthand rule (your choice).

(b) Write out the entire summation—that is, convert your Σ notation into a sum of four terms. Note that there *should not* be any x or x_i in your answer. However, you do *not* need to add up the four terms to come up with a single number.

Exercise 26.6.2. (a) Write down, using Σ notation, a Riemann sum approximating the integral

$$\int_1^4 \sin(x)\,dx$$

using $n = 6$ rectangles and using either the lefthand or righthand rule (your choice).

(b) Write out the entire summation—that is, convert your Σ notation into a sum of four terms. Note that there *should not* be any x or x_i in your answer. However, you do *not* need to add up the four terms to come up with a single number.

26.7 Some challenges

Exercise 26.7.1. How do Riemann sums for the function $f(x) = \frac{1}{x}$ from $a = 1$ to $b = t$ help you compute $\ln(t)$?

Exercise 26.7.2. Using the mean value theorem and the fundamental theorem of calculus, show that for any interval $[a, b]$ and a function f, there is some number c between a and b so that $f(c)$ is equal to the average value of f on the interval $[a, b]$.