Lecture 18

More practice problems for the midterm

18.1 Higher derivatives

Find the first, second, third, and fourth derivatives of the following functions:

(a) $f(x) = 3x^2 + 9x + 1$

(b) $f(x) = x^3 - 9$

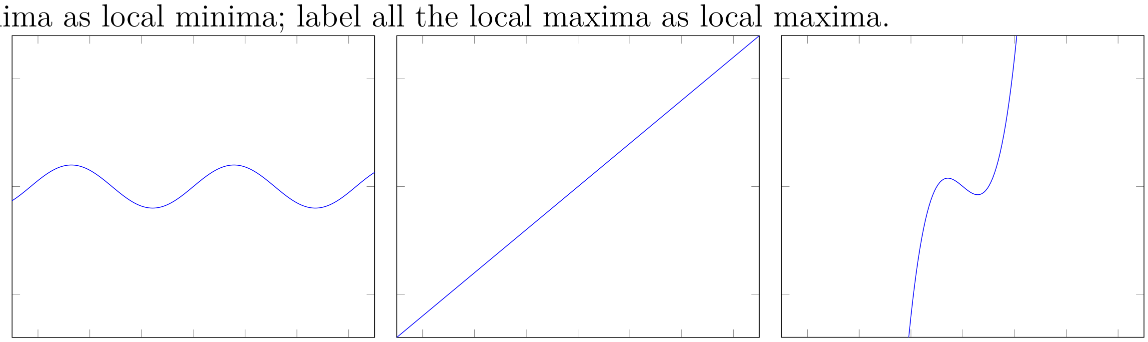
(c) $f(x) = \sin(x)$

(d) $f(x) = e^{7x}$.

(e) $f(x) = \ln x$.

(f) $f(x) = \arctan(x)$.

Exercise 18.1.1. Below are the graphs of various functions f. Label all the local minima as local minima; label all the local maxima as local maxima.



In the above pictures, what is the sign of the second derivative at the local minima?

18.2 Word problems

Exercise 18.2.1. The CDC is using a function $f(t)$ to model the number of hospitalizations due to a new disease, where t is in years, and $f(t)$ is the number of patients in the hospital at time t. For example, $f(3)$ measures the number of patients in the hospital due to the disease 3 years from now.

While the CDC does not tell us $f(t)$, they have recently told your team of researches that the *derivative* of $f(t)$ is given by the following function:

$$\frac{9-9x^2}{e^x+1}.$$

According to this, when would you expect the number of patients hospitalized due to this disease to be the greatest?

Exercise 18.2.2. You have 3,600 feet of fencing. You'd like to make a rectangular pen for your cattle to graze. (The perimeter of this rectangular pen must be 3,600 feet, because that's how much fencing you have!) What should the dimensions of the pen be to maximize the amount of area that your cattle can graze?

Exercise 18.2.3. Somebody is confused by the shape formed by the points (x, y) satisfying the equation

$$xy^2 = x^3y.$$

They want to know the slope of the tangent line to a point (x, y) on this shape. Tell them.

Assume that you're at a point where neither x nor y is equal to zero. Show them that, actually, the slope at such a point is just given by $2x$.

Exercise 18.2.4. I am filling a cone with chocolate sauce, pouring sauce into the cone at one milliliter per second. (That is, 1,000 cubic millimeters per second.) The "tip" of the cone is also the bottom of the cone, as is traditional with ice cream cones.

The cone's radius-to-height ratio is given by 1/3. So for example, for every millimeter we travel toward the tip of the cone, the radius shrinks by 1/3 of a millimeter. Given this, if I have filled the sauce to a depth of 7 millimeters from the tip, how quickly is the depth of the sauce increasing?

For this problem, it will help to know that the volume of a cone of radius r and height h is given by $\frac{1}{3}\pi r^2 h$.