

Lab Worksheet for September 21, 2021

Practice with Local Extrema and the Mean Value Theorem.

1. Find the critical points for each of the following functions.

a) $f(x) = 5x$.

b) $f(x) = 2x^2 + 3x - 12$.

c) $f(x) = x^3 + 4x^2 - 5x$.

d) $f(x) = (x-2)^3$.

2. For each of the following functions find the local minima, local maxima, and the critical points where the second derivative test is inconclusive.

a) $f(x) = xe^x$.

b) $f(x) = x^3 + 3x^2$.

c) $f(x) = x \ln(x^6)$.

d) $f(x) = 12x$.

3. Mboyo has built a wonky Ferris wheel. Your height at time t -where t is measured in hours, and you get onto the Ferris wheel at time $t = 0$, is measured by

$$h(t) = 10e^{\sin(2\pi t)}$$

where h is in meters. (So for example, when you get on initially, your height is 10 meters off the ground.)

Mboyo is so proud of his Ferris wheel that he makes you ride for seven hours.

a) You want your camera to automatically take photos from the highest points of the ride. For what time, or for what times, should you program your camera timer so that your camera automatically takes photos when you are highest on the ride?

b) Your young son is afraid of heights, and he says he only wants to open his eyes when the ride is at its lowest points. At what time, or at what times, should you tell him to open his eyes?

4. A town is emptying its reservoir. The reservoir contains 900,000 cubic meters of water. At the same time, the town's engineers know that if at any point the water is emptied at faster than 90,000 cubic meters per hour, the stress on the system may result in a huge disaster. Worse, the town does not have great models for how exactly the reservoir will empty at any given moment.

But engineering models can assume that the function (of time) representing the amount of water in the reservoir has a derivative.

a) The town asks you whether it is possible to completely empty the reservoir overnight, between 9pm and 6am, while averting disaster. What do you say? Why?

b) The town asks whether you can, using the Mean Value Theorem, guarantee that if you empty the reservoir over a space of a whole day – so between 9am and 9pm – the town can empty the reservoir while averting disaster. What do you say? Why?