Writing Assignment 2

Due Monday, September 6, 11:59 PM

This is among the "mathiest" writing assignments.

Background

We saw in class that, as h approaches zero, the expression $\sin(h)/h$ approaches 1.

You may use this fact in this assignment. You may, in fact, assume[4] the following fact as well:

$$\lim_{h\to 0} \frac{\cos(h)-1}{h} = 0.$$

Finally, you may also use the following useful fact from trigonometry. It is the angle addition formula for sine:

$$\sin(a+b) = \sin(a)\cos(b) + \sin(b)\cos(a).$$

The prompt

Using these facts, I want you to explain to me why $\sin' = \cos$.

What your submission might look like

You submission might look like a string of equalities. However, you should indicate *why* each equality you write is valid. If you are just using facts/techniques from precalculus, you may write "by precalculus facts" or "algebra." If you are using a definition of something (like the definition of a derivative), you must state "by definition of —." If you are using a fact I said you could assume, you should indicate that.

The following are two examples of what your proof might look like, for a *different* problem.

[4] I bet you can prove this, though, by multiplying the top and bottom of the fraction by $\cos(h)+1$.

Example. *Without using the power rule,* write a proof showing that

$$\frac{d}{dx}(x^2 + x) = 2x + 1.$$

Proof.

$$\begin{aligned}
\frac{d}{dx}(x^2 + x) &= \lim_{h \to 0} \frac{(x+h)^2 + (x+h) - (x^2 + x)}{h} && (1) \\
&= \lim_{h \to 0} \frac{x^2 + 2hx + h^2 + x + h - x^2 - x}{h} && (2) \\
&= \lim_{h \to 0} \frac{2hx + h + h^2}{h} && (3) \\
&= \lim_{h \to 0} 2x + 1 + h && (4) \\
&= 2x + 1. && (5)
\end{aligned}$$

The first equality is the definition of derivative. The next lines, (2) and (3) are just algebra. I can divide by h in (4) because we know $h \neq 0$ when we take this limit. The last equality is because as h approaches 0, the function $2x + 1 + h$ approaches $2x + 1$. □

Here is another way to write this proof:

Example 0.0.1. *Without using the power rule,* write a proof showing that

$$\frac{d}{dx}(x^2 + x) = 2x + 1.$$

Proof.

$$\begin{aligned}
\frac{d}{dx}(x^2 + x) &= \lim_{h \to 0} \frac{(x+h)^2 + (x+h) - (x^2 + x)}{h} && \text{definition of derivative} \\
&= \lim_{h \to 0} \frac{x^2 + 2hx + h^2 + x + h - x^2 - x}{h} && \text{algebra} \\
&= \lim_{h \to 0} \frac{2hx + h + h^2}{h} && \text{algebra} \\
&= \lim_{h \to 0} 2x + 1 + h && h \neq 0 \text{ so we can divide by } h \\
&= 2x + 1.
\end{aligned}$$

□