

## Extra Credit Assignment 4

Due Friday, September 17, 11:59 PM

This assignment takes a step back from calculus.

Recall that a rational number is a number that can be expressed as a ratio of two integers.

Recall that an *irrational* number is a number that cannot be expressed as a ratio of two integers.

Choose one of the following questions to explore, and write to me what you find, or what you are thinking. In your writing, try to constantly pursue *why* something is true, and not just whether something is true.

(I) Are there more rational numbers or irrational numbers?

(II) What are some ways you could convince somebody that a particular number is rational or irrational? For example, is  $\sqrt{7}$  rational or irrational? How do you know? How do we as a civilization know whether  $\pi$  is rational or irrational?

(III) Let's say that I have some irrational number called  $t$ . Can I approximate  $t$  using only rational numbers? That is, if somebody says "Give me a rational number  $s$  so that  $s$  and  $t$  are less than  $\epsilon$  away from each other," where  $\epsilon$  is some tiny, tiny number, can you always find such an  $s$ ?

(IV) Consider the open interval  $(0, 1)$ . (This consists of all real numbers strictly bigger than 0 but strictly less than 1.) Consider also the collection of all real numbers. Which of these has "more" elements? Be warned: Just because one thing is contained in another, it does not mean it has fewer elements!

(V) How would you explain the process of "counting" to an alien? The alien might not know what "two" or "three" mean, nor what the purpose of counting is. Can you also try to explain what we mean by the "number" of objects that a particular collection of things has? (For example, what we mean by there being thirty people in Calculus class, or infinitely many whole numbers?)