True/False practice problems for Exam I

In the exam, you will get +2 for a correct response, and 0 for an incorrect response on a True/False question. You will get +1 for writing "I don't know."

Remember: In logic, "True" means always true. "False" means "not always true," meaning the statement may be false for some examples.

State whether the statement is true or false:

(a) If $\lim_{x\to a} f(x)$ exists, then $\lim_{x\to a^-} f(x)$ and $\lim_{x\to a^+} f(x)$ both exist.

(b) If $\lim_{x\to a} f(x)$ does not exist, then $\lim_{x\to a^-} f(x)$ and $\lim_{x\to a^+} f(x)$ do not exist.

(c) If $\lim_{x\to a^-} f(x)$ and $\lim_{x\to a^+} f(x)$ exist, then $\lim_{x\to a} f(x)$ exists.

(d) If $\lim_{x\to a^-} f(x)$ and $\lim_{x\to a^+} f(x)$ exist and if they agree, then $\lim_{x\to a} f(x)$ exists.

(e) If $f'(a) = 0$, then a is either a local maximum or a local minimum.

(f) If a is an inflection point of f, then $f'(a) = 0$.

(g) If a is an inflection point of f, then $f''(a) = 0$.

(h) If a is a critical point of f, then $f'(a) = 0$.

(i) If a is a critical point of f, then $f''(a) = 0$.

(j) If $\lim_{x\to a^+} f(x) = 0$ and $\lim_{x\to a^+} g(x) = 0$, and if f and g are differentiable, we can apply L'Hopital's Rule to compute $\lim_{x\to a^+} \frac{f(x)}{g(x)}$.

(k) If $\lim_{x\to a} f(x) = 0$ and $\lim_{x\to a} g(x) = \infty$, and if f and g are differentiable, we can apply L'Hopital's Rule to compute $\lim_{x\to a} \frac{f(x)}{g(x)}$.

(l) If $\lim_{x\to a} f(x) = \infty$ and $\lim_{x\to a} g(x) = 0$, and if f and g are differentiable, we can apply L'Hopital's Rule to compute $\lim_{x\to a} \frac{f(x)}{g(x)}$.

(m) If $\lim_{x\to a^+} f(x) = \infty$ and $\lim_{x\to a^+} g(x) = \infty$, and if f and g are differentiable, we can apply L'Hopital's Rule to compute $\lim_{x\to a^+} \frac{f(x)}{g(x)}$.

(n) When testing for absolute maxima and minima for f along an interval $[a, b]$: If f is differentiable, then one need only check the values of $f(x)$ when x is a critical point or when $x = a$ or when $x = b$.

(o) The second derivative test tells us (at least) that if $f'(x) = 0$ and $f''(x) < 0$, then x is an absolute maximum.

(p) The second derivative test tells us (at least) that if $f'(x) = 0$ and $f''(x) < 0$, then x is a local maximum.

(q) The function $f(x) = |x|$ has a derivative at $x = 0$.

(r) The function $f(x) = |x|$ has a derivative at $x = 1$.

(s) The function $f(x) = |x|$ has a derivative at $x = -1$.

(t) The function $f(x) = |x|$ has a derivative at every value of x except 0.

True/False solutions

(a) If $\lim_{x\to a} f(x)$ exists, then $\lim_{x\to a^-} f(x)$ and $\lim_{x\to a^+} f(x)$ both exist. **True.**

(b) If $\lim_{x\to a} f(x)$ does not exist, then $\lim_{x\to a^-} f(x)$ and $\lim_{x\to a^+} f(x)$ do not exist. **False. Both one-sided limits *might* exist, but the two one-sided limits will not be equal.**

(c) If $\lim_{x\to a^-} f(x)$ and $\lim_{x\to a^+} f(x)$ exist, then $\lim_{x\to a} f(x)$ exists. **False. Even if both one-sided limits exist, if they are not equal, the limit** $\lim_{x\to a}$ **does not exist.**

(d) If $\lim_{x\to a^-} f(x)$ and $\lim_{x\to a^+} f(x)$ exist and if they agree, then $\lim_{x\to a} f(x)$ exists. **True.**

(e) If $f'(a) = 0$, then a is either a local maximum or a local minimum. **False. Consider** $f(x) = x^3$ **with** $a = 0$**.**

(f) If a is an inflection point of f, then $f'(a) = 0$. **False. Consider** $f(x) = \tan(x)$ **with** $a = 0$**.**

(g) If a is an inflection point of f, then $f''(a) = 0$. **True.**

(h) If a is a critical point of f, then $f'(a) = 0$. **True.**

(i) If a is a critical point of f, then $f''(a) = 0$. **False. Consider** $f(x) = x^2$**.**

(j) If $\lim_{x\to a^+} f(x) = 0$ and $\lim_{x\to a^+} g(x) = 0$, and if f and g are differentiable, we can apply L'Hopital's Rule to compute $\lim_{x\to a^+} \frac{f(x)}{g(x)}$. **True.**

(k) If $\lim_{x\to a} f(x) = 0$ and $\lim_{x\to a} g(x) = \infty$, and if f and g are differentiable, we can apply L'Hopital's Rule to compute $\lim_{x\to a} \frac{f(x)}{g(x)}$. **False. Both limits must equal zero, or both limits must equal infinity, to apply the rule.**

(l) If $\lim_{x\to a} f(x) = \infty$ and $\lim_{x\to a} g(x) = 0$, and if f and g are differentiable, we can apply L'Hopital's Rule to compute $\lim_{x\to a} \frac{f(x)}{g(x)}$. **False. Both limits must equal zero, or both limits must equal infinity, to apply the rule.**

(m) If $\lim_{x\to a^+} f(x) = \infty$ and $\lim_{x\to a^+} g(x) = \infty$, and if f and g are differentiable, we can apply L'Hopital's Rule to compute $\lim_{x\to a^+} \frac{f(x)}{g(x)}$. **True.**

(n) When testing for absolute maxima and minima for f along an interval $[a, b]$: If f is differentiable, then one need only check the values of $f(x)$ when x is a critical point or when $x = a$ or when $x = b$. **True.**

(o) The second derivative test tells us (at least) that if $f'(x) = 0$ and $f''(x) < 0$, then x is an absolute maximum. **False.**

(p) The second derivative test tells us (at least) that if $f'(x) = 0$ and $f''(x) < 0$, then x is a local maximum. **True.**

(q) The function $f(x) = |x|$ has a derivative at $x = 0$. **False. Writing the difference quotient at $x = 0$, we see that $\lim_{h\to 0^+} \frac{f(0+h)-f(0)}{h} = 1$ while $\lim_{h\to 0^-} \frac{f(0+h)-f(0)}{h} = -1$. Thus the difference quotient does not have a limit as $h \to 0$, meaning f does not have a derivative at $x = 0$.**

(r) The function $f(x) = |x|$ has a derivative at $x = 1$. **True. The derivative is 1.**

(s) The function $f(x) = |x|$ has a derivative at $x = -1$. **True. The derivative is -1.**

(t) The function $f(x) = |x|$ has a derivative at every value of x except 0. **True. The derivative is 1 when x is positive, and -1 when x is negative.**