What follows are practice problems from Sean Corrigan's class. I would expect you to be able to do every problem except 13(f).

Math 2471 Practice

1. Fill in the blanks in the following definition: We say $\lim_{x \to c} f(x) = L$ if

   for every__________, there exists __________ such that

   if __________________________, then____________________.

   Use this definition to prove that

$$\lim_{x \to 2}(6x - 1) = 11.$$

2. Let $f(x)$ be the piecewise-defined function

$$f(x) = \begin{cases} x^4 + 1 & x \text{ is rational} \\ 3x^2 + 1 & x \text{ is irrational} \end{cases}$$

   Use the Sandwich Theorem to show that $\lim_{x \to 0} f(x) = 1$.

3. Use the Sandwich Theorem to show that $\lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right) = 0$.

4. For the function $f(x) = \dfrac{x^2 - 3x - 10}{x^2 - 4x - 12}$, compute the following limits.
   (a) $\lim_{x \to 5} f(x)$
   (b) $\lim_{x \to -2} f(x)$
   (c) $\lim_{x \to 6} f(x)$

5. (a) State the formal definition of derivative: for a function $f(x)$, we define

$$f'(x) =$$

   (b) Use the formal definition of derivative to find $f'(x)$ when $f(x) = \dfrac{1}{x}$.

6. Use the Intermediate Value Theorem and the Mean Value Theorem to show that the equation $2x + \cos(x) = 0$ has exactly one solution.

7. Use the chain rule to find the derivative of the function $f(x) = \tan\left(\sin(x^2)\right)$.

8. Compute $\dfrac{dP}{dx}$ for the following functions $P(x)$.
   (a) $P(x) = \sec(x^2) \cdot \tan(x^3)$
   (b) $P(x) = (x^{-3} + x^{-1}) \cdot (7x^{3/2} - 5x + 9)$

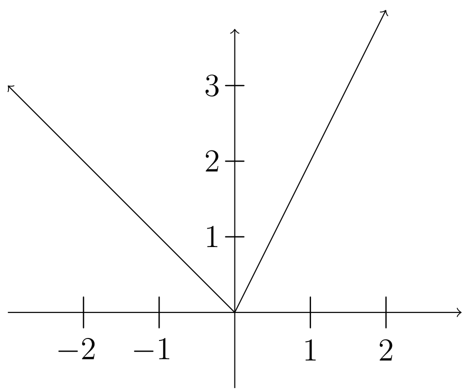
(c) $P(x) = \sqrt{x} \cdot \cos(x^2 + 1)$

9. Compute $\dfrac{dQ}{dx}$ for the following functions $Q(x)$.

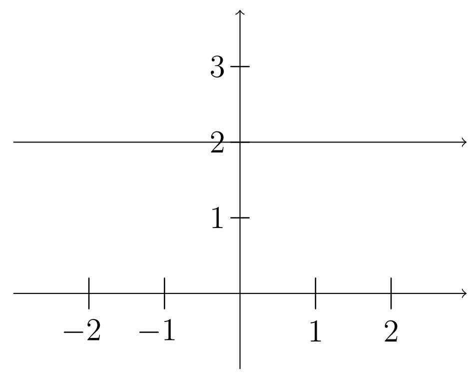
(a) $Q(x) = \dfrac{\tan(x)}{4^x}$

(b) $Q(x) = \dfrac{\arctan(e^x)}{x}$

10. Pictured here are the graphs of two functions f and g.



$f(x)$



$g(x)$

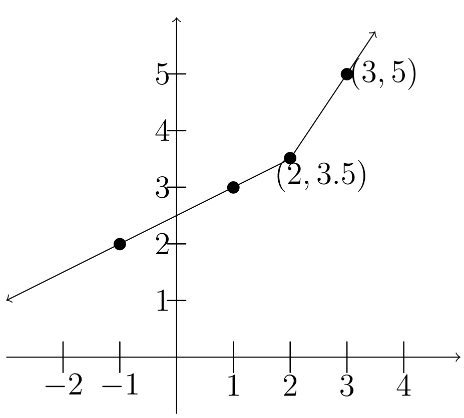
For the functions $(f \cdot g)(x) = f(x) \cdot g(x)$ and $(f \circ g)(x) = f(g(x))$, find the following derivatives.

(a) $(f \cdot g)'(1)$

(b) $(f \cdot g)'(-1)$

(c) $(f \circ g)'(1)$

11. For the one-to-one function f whose graph is pictured below, compute $(f^{-1})'(3)$.



12. A particle is moving in a straight line, and its distance from the starting point is given by the function $s(t) = t^3 + t^2$, where s is in meters and t is the time in seconds.
    (a) Find the average velocity of the particle on the interval $[1, 2]$. Give units.
    (b) Find the instantaneous velocity of the particle at the time $t = 1$. Give units.
13. Let $f(x) = x^3 - 5x + 2$.
    (a) Find the average rate of change of f on the interval $[0, 2]$.
    (b) Find the equation of the secant line through the graph of f on the interval $[0, 2]$.
    (c) Find the instantaneous rate of change of f at $x = 0$.
    (d) Find the equation of the tangent line to the graph of f at $x = 0$.
    (e) Use the equation of the tangent line to approximate $f(.1)$.
    (f) Starting with the guess $x = .2$, use Newton's Method to find a solution to the equation $f(x) = 0$ near $x = .2$.
14. A rectangle has length that is increasing at a rate of 2 m/s and width that is decreasing at a rate of 1 m/s. Find the rate of change of the area when $l = 10$ m and $w = 12$ m. Give the units for the rate of change.
15. The radius of a sphere is increasing at a rate of 5 in/s. How fast is the surface area increasing when the radius of the sphere is 20 in?
16. A conical water tank, with base radius 2 m and height 6 m, is being filled at a rate of 1 m^3/min. Find the rate at which the water level is rising when the water is 3 m deep.
17. Find the inflection points for the function $f(x) = \arctan(x^2)$. Give both the x and y values.
18. A cylindrical can has a volume of 16π cm^3. Find the dimensions of the can (radius and height) that minimize the amount of material used.
19. A box with square base and open top is to be constructed from two kinds of material – the material for the bottom of the box costs \$4/cm^2 and the material for the sides costs \$7.29/cm^2. If the box is going to have volume 25 cm^3, find the dimensions that minimize the cost of production.
20. The "kampyle of Eudoxus" is the curve

    $$y^2 = 5x^4 - x^2.$$

    The point $(-1, 2)$ is on the curve. Give the equation of the tangent line to the curve at that point.