Lecture 25

More practice!

25.1 Taking antiderivatives

Compute the following indefinite integrals. Many are taken from Guichard's textbook.

Exercise 25.1.1. $\int 25 - x^2\,dx$

Exercise 25.1.2. $\int (1-t)^9\,dt$

Exercise 25.1.3. $\int (x^2+1)^2\,dx$

Exercise 25.1.4. $\int x(x^2+1)^{100}\,dx$

Exercise 25.1.5. $\int \frac{1}{(1-5t)^{1/3}}\,dt$

Exercise 25.1.6. $\int \sin^3 x \cos x\,dx$

Exercise 25.1.7. $\int x\sqrt{100-x^2}\,dx$

Exercise 25.1.8. $\int \frac{x^2}{\sqrt{1-x^3}}\,dx$

Exercise 25.1.9. $\int \cos(\pi t)\cos(\sin(\pi t))\,dt$

Exercise 25.1.10. $\int \frac{\sin x}{\cos^3 x}\,dx$

Exercise 25.1.11. $\int \tan x\,dx$

25.2 Computing definite integrals

Exercise 25.2.1. Evaluate

$$\int_1^4 \frac{3}{x^2}\,dx$$

Exercise 25.2.2. Evaluate

$$\int_0^\pi \sin^5(3x)\cos(3x)\,dx$$

Exercise 25.2.3. Evaluate

$$\int_1^{e^2} \frac{1}{x}\,dx$$

Exercise 25.2.4. Evaluate

$$\int_1^8 \frac{3x^2+2}{\sqrt{x}}\,dx$$

Exercise 25.2.5. Evaluate

$$\int_0^{2\pi} 8\cos(x)\,dx$$

Exercise 25.2.6. Evaluate

$$\int_1^8 2x+10\,dx$$

25.3 Areas between curves

Exercise 25.3.1. Find the area between the graphs of $x^2+2x-10$ and $4x-7$.

Exercise 25.3.2. Find the area between the three curves $y=x$ and $y=7x$ and $x=1$. (You may want to draw a picture.)

Exercise 25.3.3. Find the area between the graphs of x^3 and x^2.

25.4 Average values

Exercise 25.4.1. Find the average value of $f(x)=3+2x^2$ on the interval $[0,\sqrt{3}]$.

Exercise 25.4.2. An object attached to a (horizontally aligned) spring moves with velocity $v(t)=\sin(t)$.

(a) What is the average velocity of this object over the interval $[0, 2\pi]$?

(b) What is the average velocity of this object over the interval $[0, \pi]$?

Exercise 25.4.3. The number of new infections per day at the beginning of an outbreak can be modeled by the function $f(t) = e^t$, where t is in days and $f(t)$ is in units of (new) infections per day.

(a) At day 10, how many new infections are arising per day? (You can use a calculator if you want a decimal answer.)

(b) Between day 0 and day 10, on average, how many new infections have there been per day? (You can use a calculator if you want a decimal answer.)

25.5 Word problems

Exercise 25.5.1. A particle moves with velocity function $v(t) = -t^2 + 3x - 2$. Find the displacement (the signed distance between the starting and ending point) of the particle over the time interval $[-2, 3]$.

Exercise 25.5.2. An epidemiologist models that by day t of a pandemic, her county will have accumulated

$$\int_0^t 10e^{5x}\,dx$$

infections total. According to this, how many new infections per day will her county be seeing at the moment $t = 5$? (Hint: Fundamental Theorem.)

25.6 Some challenges

Exercise 25.6.1. How do Riemann sums for the function $f(x) = \frac{1}{x}$ from $a = 1$ to $b = t$ help you compute $\ln(t)$?

Exercise 25.6.2. Using the mean value theorem and the fundamental theorem of calculus, show that for any interval $[a, b]$ and a function f, there is some number c between a and b so that $f(c)$ is equal to the average value of f on the interval $[a, b]$.