

Lecture 24

Average values

Definition 24.0.1. Let f be a function and choose two real numbers a and b such that $a < b$. Then *the average value* of f over the interval $[a, b]$ is defined to be

$$\frac{\int_a^b f(x) dx}{b - a}.$$

Of course, the above is equal to

$$\frac{1}{b - a} \int_a^b f(x) dx.$$

Example 24.0.2. A bird travels. The velocity at time t is given by the function $v(t) = t^2$ kilometers per second. What is the average velocity with which the car was traveling between $t = 0$ and $t = 3$?

Solution. We must compute

$$\frac{1}{b - a} \int_a^b f(t) dt = \frac{1}{3 - 0} \int_0^3 t^2 dt \tag{24.0.1}$$

$$= \frac{1}{3} \left(\frac{1}{3} t^3 \Big|_0^3 \right) \tag{24.0.2}$$

$$= \frac{1}{3} \left(\frac{1}{3} ((3)^3 - (0)^3) \right) \tag{24.0.3}$$

$$= \frac{1}{9} (3 \times 9) \tag{24.0.4}$$

$$= 3. \tag{24.0.5}$$

The average speed of the verb is 3 kilometers per second.

24.1 Practice day!

The rest of today will just be practicing integrals.

Compute the following indefinite integrals and (where bounds are indicated) definite integrals. They are taken from Guichard's textbook.

Exercise 24.1.1. $\int (1 - t)^9 dt$

Exercise 24.1.2. $\int (x^2 + 1)^2 dx$

Exercise 24.1.3. $\int x(x^2 + 1)^{100} dx$

Exercise 24.1.4. $\int \frac{1}{(1-5t)^{1/3}} dt$

Exercise 24.1.5. $\int \sin^3 x \cos x dx$

Exercise 24.1.6. $\int x\sqrt{100 - x^2} dx$

Exercise 24.1.7. $\int \frac{x^2}{\sqrt{1-x^3}} dx$

Exercise 24.1.8. $\int \cos(\pi t) \cos(\sin(\pi t)) dt$

Exercise 24.1.9. $\int \frac{\sin x}{\cos^3 x} dx$

Exercise 24.1.10. $\int \tan x dx$

Exercise 24.1.11. $\int_0^\pi \sin^5(3x) \cos(3x) dx$