

Extra Credit Assignment 6

Due Friday, October 2, 11:59 PM

Here is a kind of “related rates” problem, but quite different. Usually in related rates problems, we know how a function depends on other factors (often via a formula), and we can deduce the rate of change of the function in terms of those factors.

But here is a different kind of problem: Suppose you know that the rate of change of something is *proportional* to how the quantity of that something. (And this proportion does not change with time.) In other words,

$$f'(t) = Cf(t)$$

for some constant C . (Somebody might demand that C equal 5, or 0, or -1, or $1/\pi$; each such choice results in a different equation.)

What kind of function could $f(t)$ be? Can you write down all functions f that satisfy the above equation? Can you write down an f that satisfies the above equation for $C = 5$? That is, can you write down an f so that

$$f'(t) = 5f(t)?$$

What if you change C ? And can you write down *all* such functions? How can you convince me that you’ve written down all of them?