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ON THE WORK OF JOHN THOMPSON

by R. BRAUER

It is an honor to be called upon to describe to you the brilliant work for which John Thompson has just been awarded the Fields medal. The pleasure is tempered by the feeling that he himself could do this job much better. But perhaps I can say some things he would never say since he is a modest person.

The central outstanding problem in the theory of finite groups today is that of determining the simple finite groups. One may say that this problem goes back to Galois. In any case, Camille Jordan must have been aware of it. Important classes of simple groups have been constructed as well as some individual types of such groups. French mathematicians, Galois, Jordan, Mathieu, Chevalley, have been the pioneers in this work. In recent years, mathematicians of many different countries have joined. However, the general problem is unsolved. We do not know at all how close we are to knowing all simple finite groups. I shall not discuss the present situation of the problem since this will be the topic of Feit's address at this congress. I may only say that up to the early 1960's, really nothing of real interest was known about general simple groups of finite order.

I shall now describe Thompson's contribution. The first paper I have to mention is a joint paper by Walter Feit and John Thompson and, of course, Feit's part in it should not be overlooked. Here, the authors proved a famous conjecture, to the effect that all non-cyclic finite simple groups have even order. I am not sure who was the first to observe this. Fifty years ago this was already referred to as a very old conjecture. While it was usually mentioned in courses on algebra, it is only fair to say that nobody ever did anything about it, simply because nobody had any idea how to get even started. It was not even clear that the whole problem made much sense. Was the role of the prime 2 simply a little accident; did 2 play an entirely exceptional role, or were there properties of other prime divisors of the group order which bore at least some resemblance to those of 2? It was only after the Feit-Thompson paper that one could be sure that the whole question has been a reasonable one.

Thompson's work which has now been honored by the Fields medal is a sequel to this first paper. In it, he determines the minimal simple finite groups, this is to say, the simple finite groups, whose proper subgroups are solvable. Actually, a more general problem is solved. It suffices to assume that only certain subgroups, the so-called local subgroups, are solvable. These are the normalizers of subgroups of prime power order larger than I.

These results are the first substantial results achieved concerning simple groups. A number of important corollaries show that one is now able to answer questions on finite groups which had been completely out of reach before. I mention one: a finite groups is solvable, if and only if every subgroup generated by two elements is solvable. You only have to try to prove this yourself if you want to see how deep the result lies.

Both investigations are very long and complicated and their logical structure is extremely intricate. Unfortunately, I cannot even give you a vague idea of the methods. Reading the papers, one reaches stages repeatedly that one feels caught in a hopeless situation, in an abyss from which there is no escape. Then, miraculously, a way out appears, an amazing turn, which saves us. A famous 19-th century mathematician once remarked that group theory could be done by people who did not know much else of mathematics. There may be some truth in this, but I think, this was not meant in a very nice way. However I believe it was overlooked that if you work in a field where you have few tools, you have to create your own tools. In order to reach positive achievements, mathematical imagination must replace knowledge from other fields.

There is other important work of Thompson in group theory which I cannot discuss here. His methods have already been used successfully by other mathematicians who have developed some of them further. In this way, Thompson has had a tremendons influence. Since he first appeared at the International Congress in Stockholm eight years ago, finite group theory simply is not the same any more.

Let me finish with a personal remark. One reaches a point in life where one wonders what one still expects of life, what one would still like to see happen. This applies to events in Mathematics too. I have passed the point I mentioned. I like to say that I would like to see the solution of the problem of the finite simple groups and the part I expect Thompson's work to play in it. Quite generally, I would like to see to what further heights Thompson's future work will take him. I feel I should also say the same about the three other Fields medallists.

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