Math 122 Section, 9/18

1 Cyclic Groups

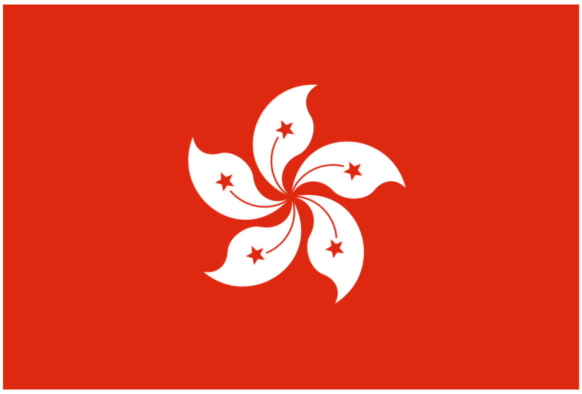
1.1 All cyclic groups are abelian

Proof. Call the cyclic group G, which is generated by the element g. Then $\forall x, y \in G, \exists m, n$ such that $g^m = x$ and $g^n = y$. Therefore

$$xy = g^m g^n = g^{m+n} = g^n g^m = yx$$

Therefore G is abelian.

1.2 examples

1. $\mathbb{Z}/n\mathbb{Z}$
2. $\mathbb{Z}^\times/p\mathbb{Z}$
3. nth roots of unity
4. $\mathbb{Z}$
5. 

□

2 Product Groups

1. Klein four-group $=C_2 \times C_2$

1	a
b	ab

   Smallest non-cyclic group

2. If $\{z \in T \subset \mathbb{C} : |z| = 1\}$, then $\mathbb{C}^\times = \mathbb{R}^\times \times T$