MATH 122 SYLLBAUS
HARVARD UNIVERSITY MATH DEPARTMENT, FALL 2014

INSTRUCTOR: HIRO LEE TANAKA
UPDATED THURSDAY, SEPTEMBER 4, 2014

Location: Harvard Hall 102
E-mail: hirohirohiro@gmail.com
Class Meeting Time: M., W., F., at 12 PM
What you should call me: Hiro

Course Content

This course is an introduction to groups and rings, which are foundational concepts in modern mathematics. If time allows (which it probably won't) we will sneak a peek at cryptography or Noether's Theorem. Along the way, we will also deepen our understanding of linear algebra and the role of structures on vector spaces.

Prerequisites: I will assume you are all familiar with vector spaces, linear transformations, and matrices. Basic familiarity with mathematical proof is necessary; I recommend that you have taken at least one proof-based class before.

Office hours and Course Assistants

Hiro's Office: Science Center 341 (in the back of the Birkhoff Math Library)
Office Hours: Wednesdays, 2 PM - 4 PM and Thursdays, 12:30 PM - 2 PM.
The first Office Hours will take place on Wednesday, September 10th.

These are times I will be in my office. I set them aside to have time to interact with you—to help, to expand, to deepen your experience in the class. Because I am by the library, my door will often be closed. But knock, and I will answer.

Your course assistants are Theo McKenzie, Kevin Yang, and Octave Dragoi. Once the course enrollment stabilizes, we will schedule appropriate times for a once-a-week recitation section. CAs will also be holding separate office hours from mine.

Examinations

There will be an in-class exam on Wednesday, October 15. There will also be a take-home midterm due Tuesday, December 2 (the day before Reading Period begins). The Final Exam will not be a take-home exam, and will be given whenever Harvard mandates. (Our exam group is 11.)

Collaboration and plagiarism policy

I strongly encourage all of you to collaborate. Please do so. If you do, you must indicate clearly on every assignment that you have collaborated, and indicate with whom. However, *write* solutions on your own. It is fine to think through problems and find solutions with each other, but **when it comes to the act of writing it all up, you must do so without assistance from another**. This is because the act of solving something and writing a mathematical proof are two different skills, and I want you to also hone the latter. As an extreme anti-example, copying and pasting solutions/proofs will not be tolerated. To reiterate, you may not *write* solutions together.

Finally, note that asking for a solution on Stackexchange, Quora, or Yahoo Answers is *not* considered collaboration in this class. I strongly discourage you from handing in any solution obtained by searching through, or asking on, a website like the ones listed above.

Grading

We will use five metrics/components to determine your grade in this class, with each metric given equal weight (20 percent) in the final figure. The five components are:

(1) Homework (typically once a week, due Mondays)
(2) Make-up
(3) Midterm 1
(4) Midterm 2
(5) Final Exam.

The **Make-up** component may not be a typical component for other classes, so I will explain this. You will most likely submit incorrect homework or exam solutions at some point. You **must** come to an office hours (or make an appointment, with either me or the CA's) at some point to explain what the correct solution. This must be done less than three weeks after the return of the homework assignment.

Textbook and Course Website

We will not use a textbook in this class. Notes will be posted on the course website after lectures, and there are several online sources you can use to cross-reference:

(1) Judson's free text: `http://abstract.ups.edu/download.html`
(2) Milne's various notes: `http://www.jmilne.org/math/CourseNotes/index.html`
(3) Beachy's various notes: `http://www.math.niu.edu/~beachy/`
(4) Conrad's notes on various topics: `http://www.math.uconn.edu/~kconrad/blurbs/`
(5) Dick Gross's lectures: `http://www.extension.harvard.edu/open-learning-initiative/abstract-algebra`

You might also consider purchasing Artin's Algebra, which is a great introduction to the ideas of our class. Other common texts include Dummit and Foote, and Lang's Undergraduate Algebra. These, however, are not legally available for free.

The course website is `http://math.harvard.edu/~hirolee/index.php?pageID=2014-122`. The iSite will host identical content. At both websites, you will find the homework sets.

1. Topics covered

Groups:

- Definition of groups.
- Examples: $\mathbb{Z}, \mathbb{R}, \mathbb{C}, S_n, D_{2n}, GL_n$, Elliptic curves*, free groups
- Group homomorphisms, subgroups, kernels
- Generators and relations. Growth of groups*, Cayley graphs*.
- Order, cyclic groups
- Group actions, stabilizers, orbits
- Conjugation action, normal subgroups, cosets, quotient groups
- Hyperbolic geometry of Poincare disk and surfaces of high genus*.
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Outline (very tentative)

Roughly 40 lectures.

(1) Wed., Sept 3. Intro. Groups as symmetries, rings as functions. The geometry of algebra beyond Cartesian geometry: Quotient rings and functions on polynomial loci, groups actions, symmetries as spaces themselves.
(2) Define groups. Subgroups. Group homomorphisms. Group isomorphisms. Examples. Integers, Real numbers, matrices, real numbers under multiplication, complex numbers. The circle. Non-commutative examples: Symmetric groups. Matrix groups. Determinant, norm from $\mathbb{C}^\times \to \mathbb{R}^\times$, exponential map.
(3) Brief non-examples: Monoids, non-negative numbers, all matrices under multiplication, et cetera. Group actions on sets and spaces. Orbits, stabilizers. (When does an orbit space retain a property of the total space?).
(4) Wed., Sept 10. Examples of actions: Symmetric group acting on sets, matrices acting on each other, on spheres, on vector spaces. Groups acting on themselves. **Homework 1 Due. Product groups, automorphisms, some linear algebra**
(5) Orbits, stabilizers. Cosets and quotient groups. $\mathbb{Z}/n\mathbb{Z}$ as an example. Constructing quotient groups using a magic property which we will later call "normal."
(6) Normal subgroups. First isomorphism theorem. Modular groups.
(7) Wed., Sept 17. Elliptic curves. **Homework Two Due. Quotient groups, kernels.**
(8) Group actions on sets. Counting formulas. Orders of group elements. The cyclic groups and $\mathbb{Z}$.
(9) Wed., Sept 24. Conjugation action. Counting formulas continued. Conjugation classes.
(10) Sylow's Theorems, I. Statements without proof. **Homework three due. Baby Sylow theorems, Lagrange's theorems.**
(11) Sylow's Theorems, II
(12) Wed., Oct 1. Sylow's Theorems, III
(13) Studying symmetric groups, I. **Homework four due. Gaps in in-class proofs of Sylow's theorems. Applications.**
(14) Studying symmetric groups, II.
(15) Wed., Oct 8. Simplicity of A_n for $n \geq 5$. Hand-waving of why polynomials of degree ≥ 5 may not be solvable.
(16) Wrap-up of groups, cryptography if possible. **Homework five due. Two more applications of Sylow's theorems. Solvable groups.**
(17) Extra classes in case we need it.
(18) Extra classes in case we need it.
(19) Extra classes in case we need it.

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END OF GROUPS. Should roughly end on October 10. No class on October 13th (Columbus Day.) Midterm on October 15.

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Beginning of rings. Should roughly begin on October 20th.

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(20) Introduction to rings. Definition, Examples: Polynomial rings, group rings, matrix rings.
(21) Fields, k-algebras, Polynomial rings over fields, division algorithm.
(22) Left and right ideals. Quotient rings. Examples: Quotients of polynomial rings. Modular arithmetic. **Homework six due. Rings $\mathbb{Z}/n\mathbb{Z}$. Direct sums of rings. Isomorphisms between one-dimensional rings.**
(23) Prime ideals, fields revisited, irreducible polynomials, division algorithm.
(24) Introduction to modules: Rings act on objects living on a space: Functions on themselves, functions on vector fields. Definition of modules, submodules, quotient modules.
(25) Module maps, passing to commutative rings: Module maps as a module. Dual modules.
(26) Tensor products and matrices. **Homework seven due. Computing quotient modules, localizing modules.**
(27) Diagonalizing maps between free modules over Euclidean domains, I. (12.4 of Artin)
(28) Diagonalizing maps between free modules over Euclidean domains, II. (12.4 of Artin) **Homework eight due. Free and projective modules.**
(29) Classification of finitely generated abelian groups. (12.6 of Artin.)
(30) Modules over $k[t]$ for k a field. (12.7 of Artin.)
(31) Modules over $\mathbb{C}[t]$, Jordan canonical form. (12.7 of Artin.)
(32) Linear algebra: Generalized eigenvalues
(33) Linear algebra: Minimal polynomials and characteristic polynomials over algebraically closed fields
(34) Linear algebra: Spectral decomposition

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End of rings. Should roughly end on November 24th, Thanksgiving Break.

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Beginning of representation theory. Should begin on December 1st.

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End of representation theory. Should end on December 12th.
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Finals Dec 12 - 16.
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Wednesday, September 3	Groups, homomorphisms, isomorphisms, subgroups, examples of groups
Friday, September 5	Examples of group homomorphisms, monoids, symmetric groups, linear groups
Monday, September 8	Equivalence relations, cosets, $\mathbb{Z}/n\mathbb{Z}$.
Wednesday, September 10	Normal subgroups, quotient groups, first isomorphism theorem.
Friday, September 12	Second isomorphism theorem, examples
Monday, September 15	Third isomorphism theorem, examples
Wednesday, September 17	Symmetric groups and permutations
Friday, September 19	Elliptic curves
Monday, September 22	Rings and ring homomorphisms, examples.
Wednesday, September 24	Ideals, commutative rings, group rings, polynomial rings
Friday, September 26	Modules, sections of bundles as examples. Module maps.
Monday, September 29	Submodules, quotient modules.
Wednesday, October 1	Free objects: Free groups, free commutative rings, free modules.
Friday, October 3	Direct sums and direct products of modules. Universal properties. First examples of functors.
Monday, October 6	Review of eigenvalues and eigenspaces
Wednesday, October 8	Generalized eigenvalues
Friday, October 10	
Monday, October 13	
Wednesday, October 15	
Friday, October 17	
Monday, October 20	
Wednesday, October 22	
Friday, October 24	
Monday, October 27	
Wednesday, October 29	
Friday, October 31	
Monday, November 3	
Wednesday, November 5	
Friday, November 7	
Monday, November 10	
Wednesday, November 12	
Friday, November 14	
Monday, November 17	
Wednesday, November 19	
Friday, November 21	
Monday, November 24	
Wednesday, November 26	
Friday, November 28	
Monday, December 1	
Wednesday, December 3	
Friday, December 5	
Monday, December 8	
Wednesday, December 10	
Friday, December 12	
Monday, December 15	
Wednesday, December 17	